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This booklet, one of a series, has been developed for this project. A Program for Mathematically Underdeveloped Pupils. A project team, including inservice teachers, is being used to write and develop the materials for this program. The materials developed in this booklet are based on activities involving (1) similar geometric figures, (2) similar triangles, (3) classification of triangles, (4) constructing triangles and similar triangles, and (5) finding the missing length of similar polygons. (RP)

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ESEA Title III
PROJECT MATHEMATICS

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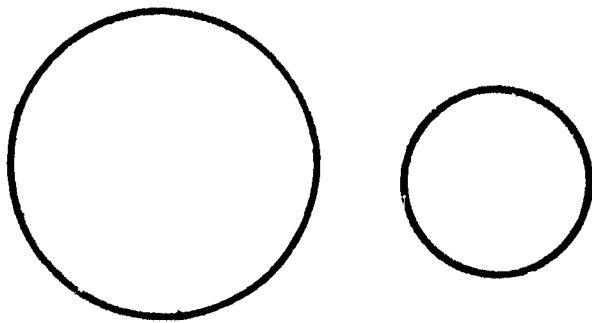
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TABLE OF CONTENTS

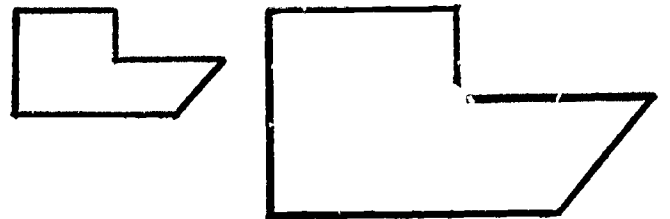
	PAGE
Similar Geometric Figures	1
Activities--Similar?	2
Activities--Drawing	5
Activities--Matching	7
Similar Triangles	8
Pairings or Correspondences of Lengths of Sides of Similar Triangles	9
Activities--Completion of Tables	10
Activities--Pairing	20
Classification of Triangles	23
Activities--Classifying Triangles	25
The Missing Side of Similar Triangles	28
Activities--Missing Length	29
"Construction" of a Triangle	35
Activities--Triangle "Construction"	36
"Construction" of Similar Triangles and Activities	38
Similar Polygons	41
Activities--Missing Lengths	42

SIMILARITY

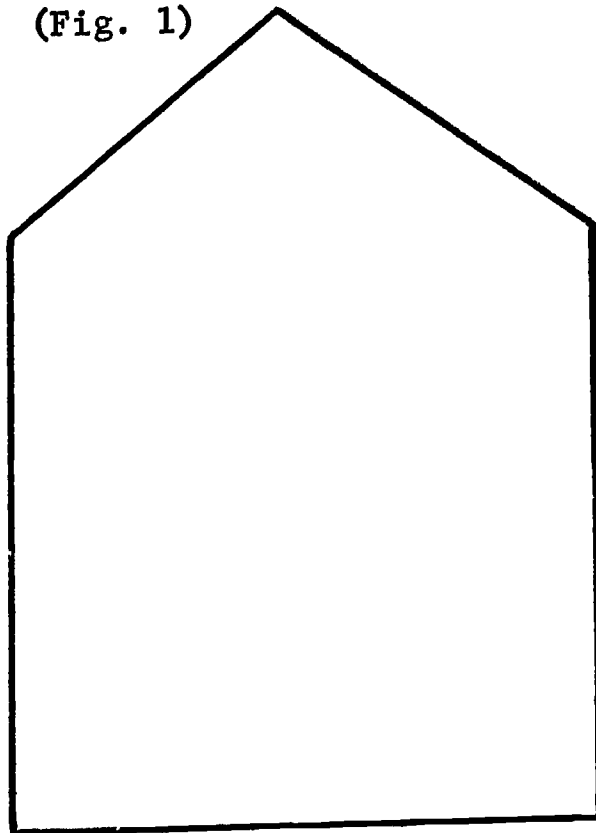
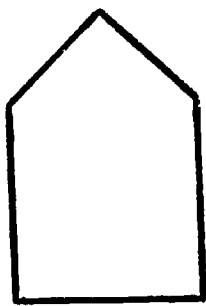
Similar geometric figures are geometric figures which have the same shape. Here are some examples of similar figures. (You can see that two similar geometric figures have either the same shape and size or one is an enlargement of the other.)



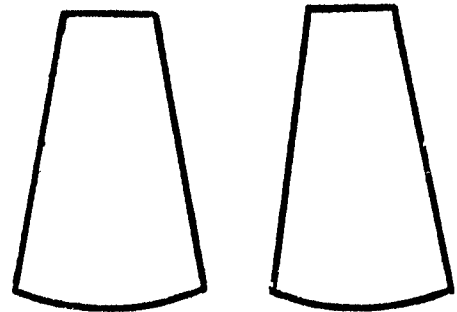
(Fig. 1)



(Fig. 2)



(Fig. 3)

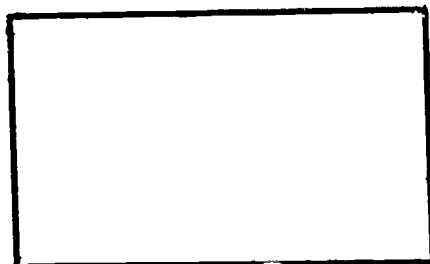
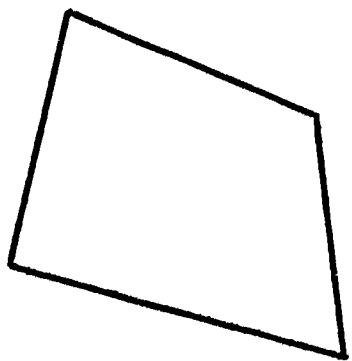


(Fig. 4)

Activities

Keep in mind that similar geometric figures are geometric figures which have the same shape but not necessarily the same size. Which of the following pairs of geometric figures are similar? Circle yes or no.

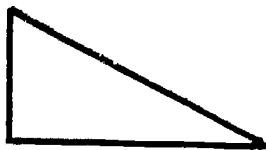
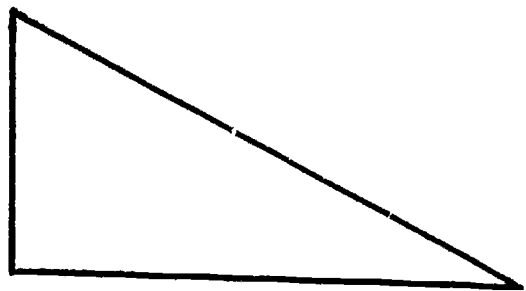
1.



Yes

No

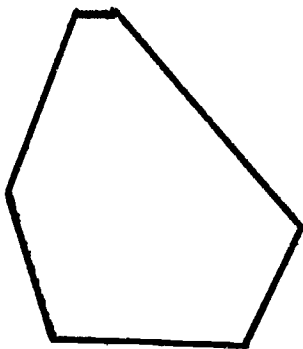
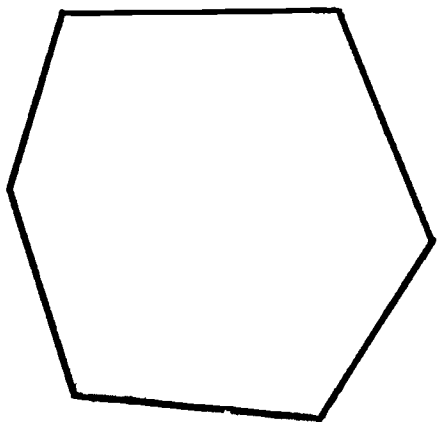
2.



Yes

No

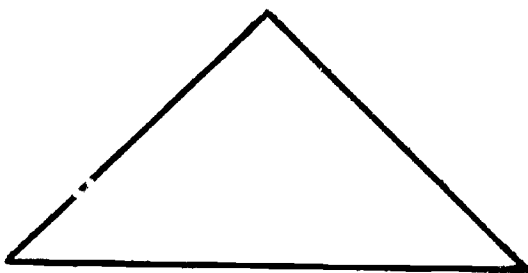
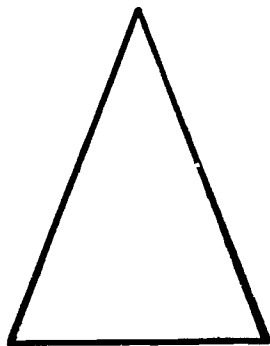
3.



Yes

No

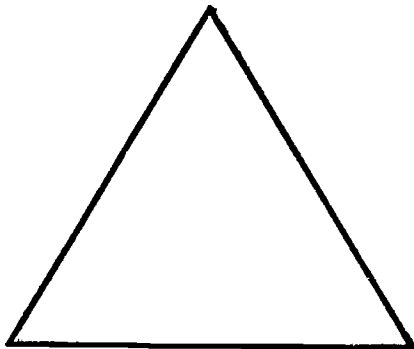
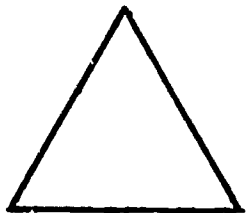
4.



Yes

No

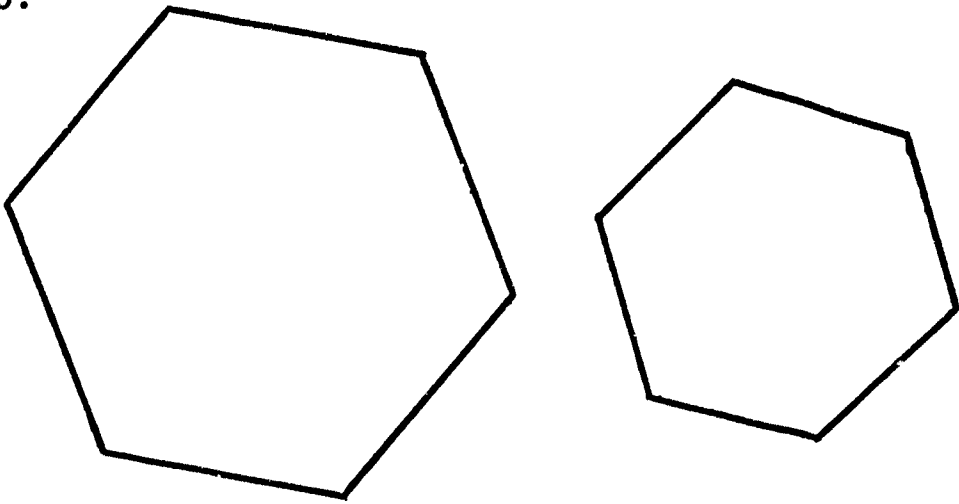
5.



Yes

No

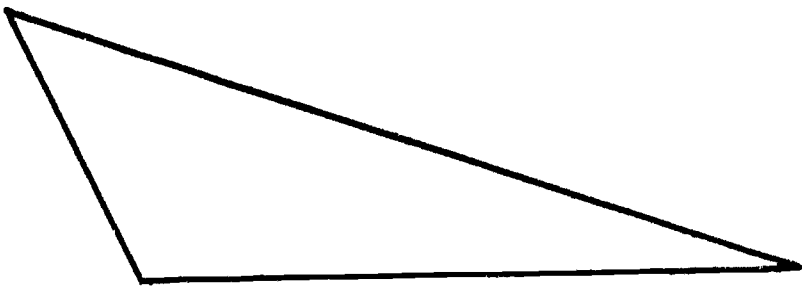
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Yes

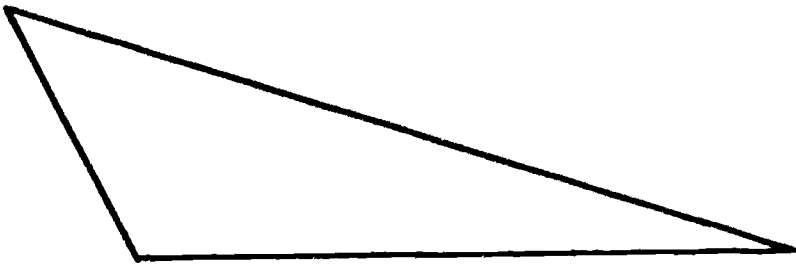
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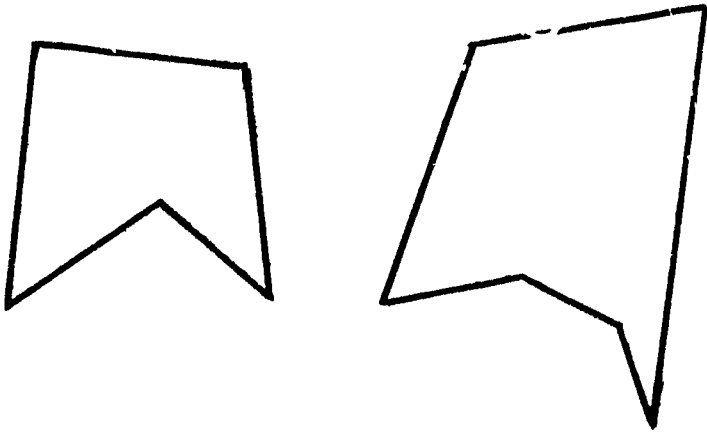


Yes

No



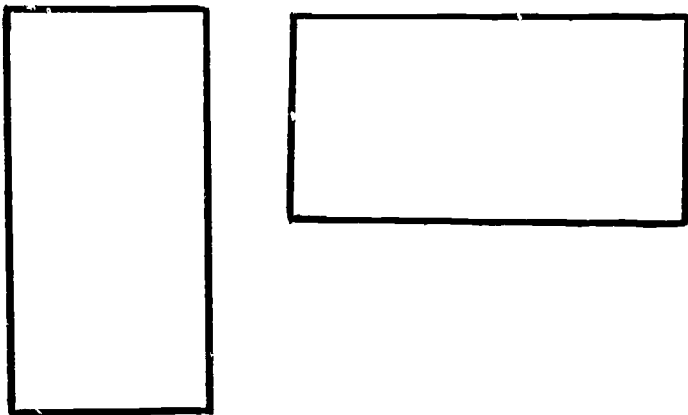
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Yes

No

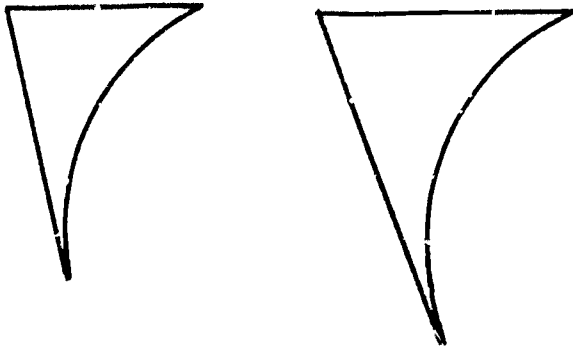
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Yes

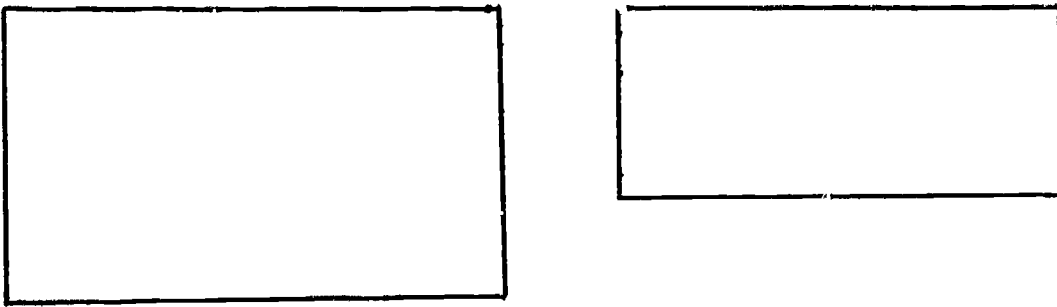
No

10.



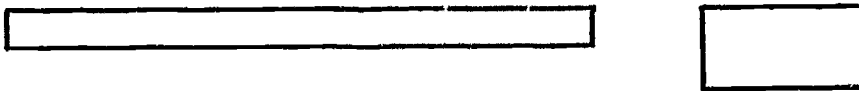
Yes No

11.



Yes No

12.

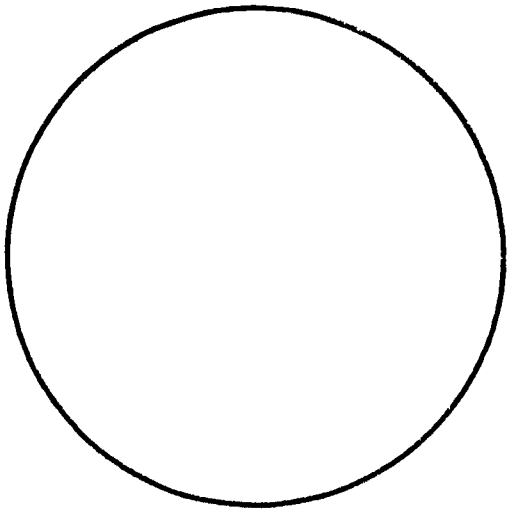


Yes No

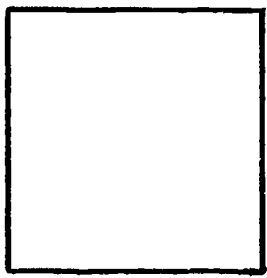
Activities

Draw a picture of a plane geometric figure which is similar to that of each figure given below. Use only your compass and ruler.

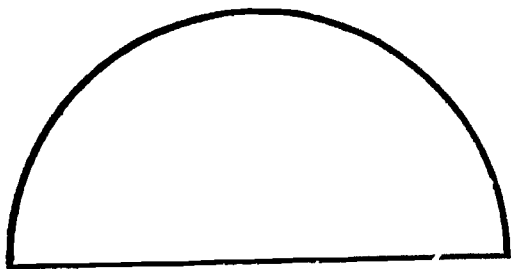
1.



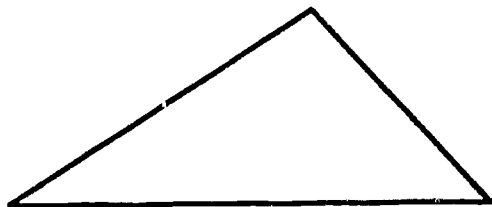
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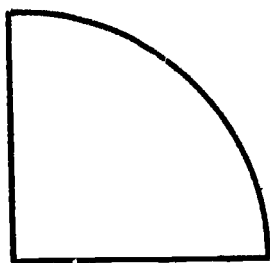
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4.



5.



6.



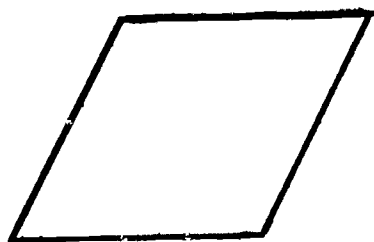
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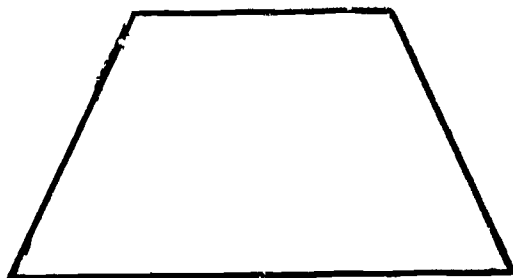
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9.



10.

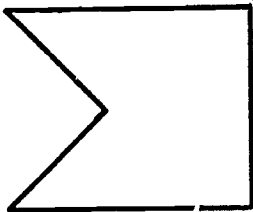


Match the similar figures in the two columns which follow by connecting them with lines. The first is an example.

1.



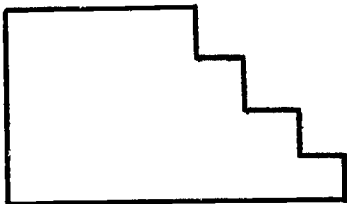
2.



3.



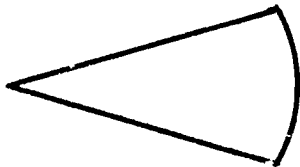
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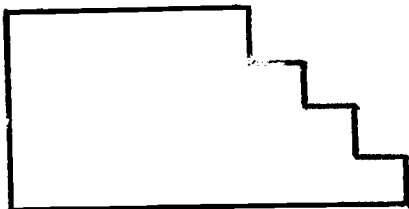
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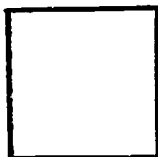
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7.



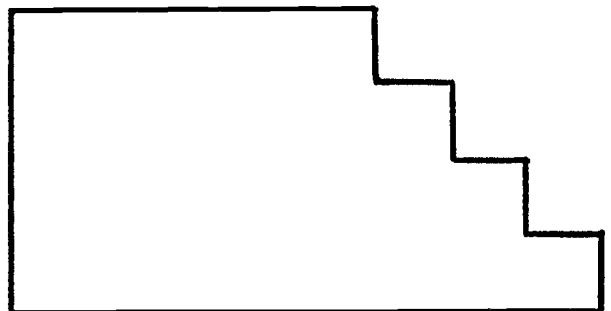
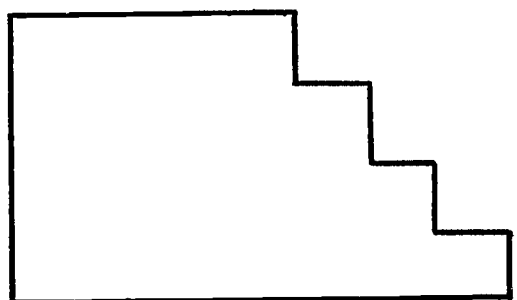
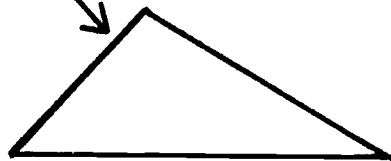
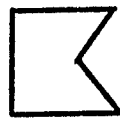
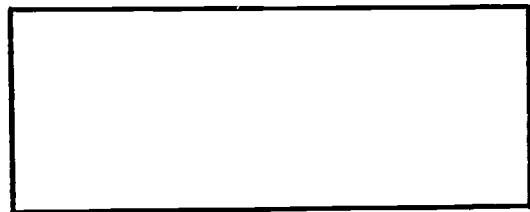
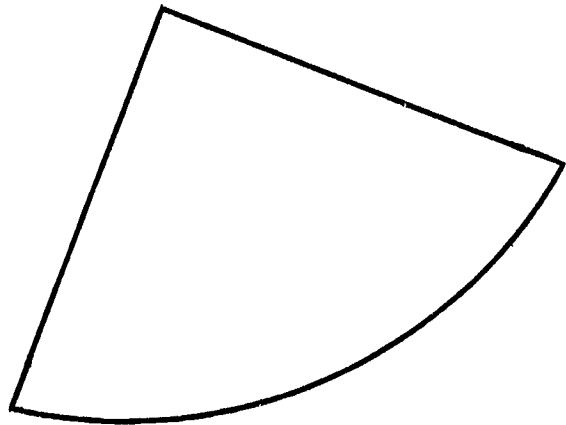
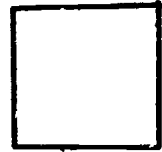
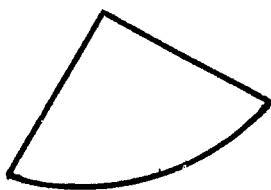
8.



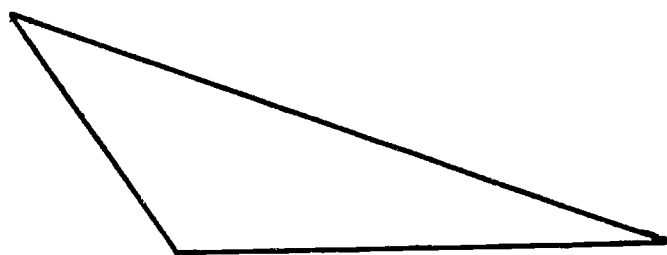
9.



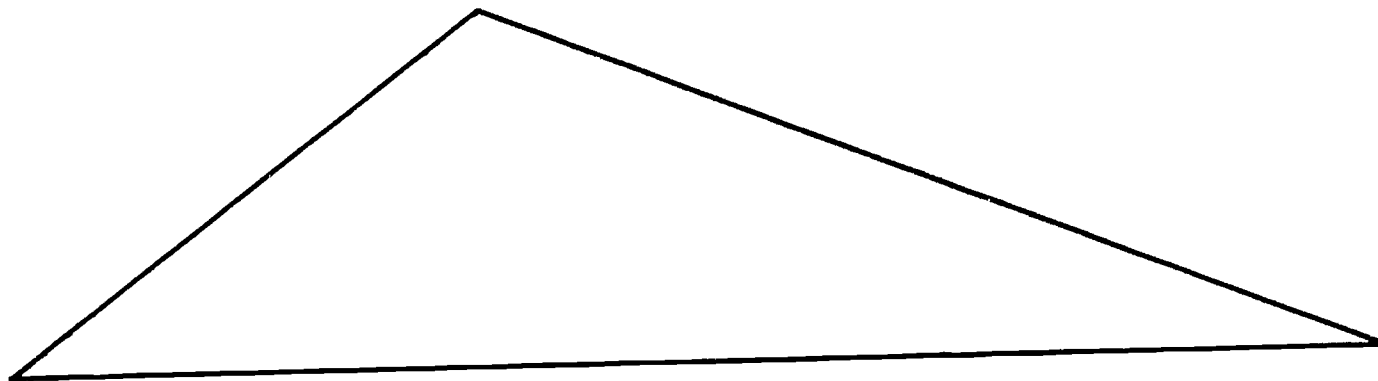
10.



Are the triangles pictured in figures 5 and 6 similar? Think before you answer. _____

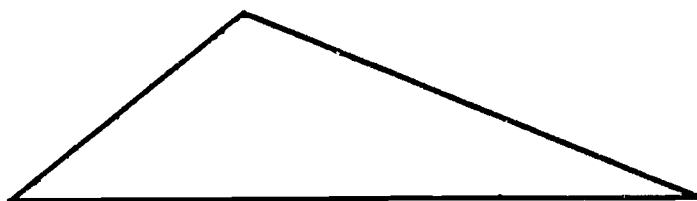


(Fig. 5)



(Fig. 6)

Figure 5 can be redrawn to look like this:



(Fig. 7)

Do you think the triangles pictured in figures 6 and 7 are similar?

_____ Then are the two triangles represented by figures 5 and 6 similar? _____

The lengths of the sides of the triangle in figure 5 are: 3 centimeters, 7 centimeters, and 5 centimeters. The lengths of the sides of the triangle in figure 6 are: 6 centimeters, 10 centimeters, and 14 centimeters.

Examine the following table. The measures of the lengths of the sides of these two triangles may be paired in six different ways.

(A pairing is usually referred to as a correspondence.)

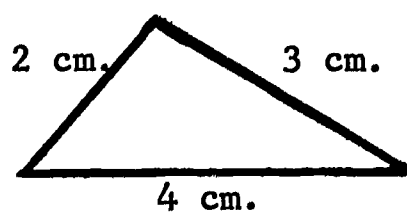
	Measures of the lengths of the sides of the triangle pictured in Figure 5	Measures of the lengths of the sides of the triangle pictured in Figure 6	Pairing or Correspondence	Fractions
1.	3	6	$3 \longleftrightarrow 6$	$\frac{3}{6}$
	7	10	$7 \longleftrightarrow 10$	$\frac{7}{10}$
	5	14	$5 \longleftrightarrow 14$	$\frac{5}{14}$
2.	3	6	$3 \longleftrightarrow 6$	$\frac{3}{6}$
	7	14	$7 \longleftrightarrow 14$	$\frac{7}{14}$
	5	10	$5 \longleftrightarrow 10$	$\frac{5}{10}$
3.	3	10	$3 \longleftrightarrow 10$	$\frac{3}{10}$
	7	6	$7 \longleftrightarrow 6$	$\frac{7}{6}$
	5	14	$5 \longleftrightarrow 14$	$\frac{5}{14}$
4.	3	10	$3 \longleftrightarrow 10$	$\frac{3}{10}$
	7	14	$7 \longleftrightarrow 14$	$\frac{7}{14}$
	5	6	$5 \longleftrightarrow 6$	$\frac{5}{6}$
5.	3	14	$3 \longleftrightarrow 14$	$\frac{3}{14}$
	7	10	$7 \longleftrightarrow 10$	$\frac{7}{10}$
	5	6	$5 \longleftrightarrow 6$	$\frac{5}{6}$
6.	3	14	$3 \longleftrightarrow 14$	$\frac{3}{14}$
	7	6	$7 \longleftrightarrow 6$	$\frac{7}{6}$
	5	10	$5 \longleftrightarrow 10$	$\frac{5}{10}$

Do you notice in 2, that the correspondence yields a set of equivalent fractions $\left\{\frac{3}{6}, \frac{7}{14}, \frac{5}{10}\right\}$? That is, $\frac{3}{6} = \frac{7}{14} = \frac{5}{10}$.

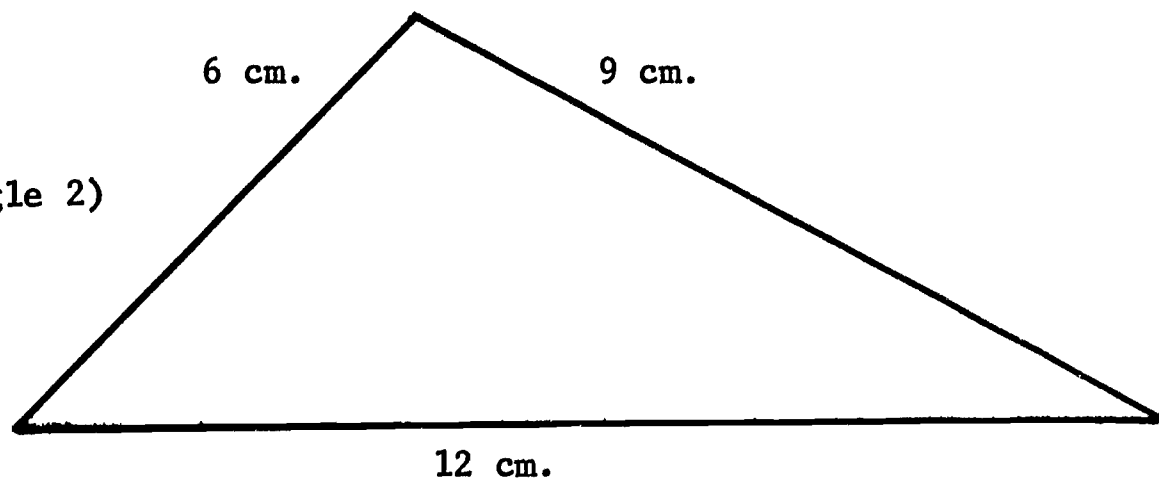
ACTIVITIES

1. The two triangles pictured here are similar. Complete the following table.

(Triangle 1)



(Triangle 2)

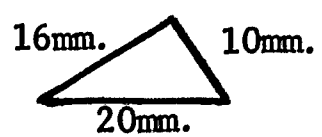


Lengths of sides of triangle 1 (in cm.)	Lengths of sides of triangle 2 (in cm.)	Correspondence or Pairing	Fractions
2	9	$2 \longleftrightarrow 9$	$\frac{2}{9}$
3	12	$3 \longleftrightarrow 12$	$\frac{3}{12}$
4	6	$4 \longleftrightarrow 6$	$\frac{4}{6}$
2	9	$2 \longleftrightarrow 9$	_____
3	6	_____	_____
4	12	_____	_____
2	12	_____	_____
3	9	_____	_____
4	_____	_____	_____
2	12	_____	_____
3	6	_____	_____
4	_____	_____	_____
2	6	_____	_____
3	_____	_____	_____
4	_____	_____	_____
2	6	_____	_____
3	_____	_____	_____
4	_____	_____	_____

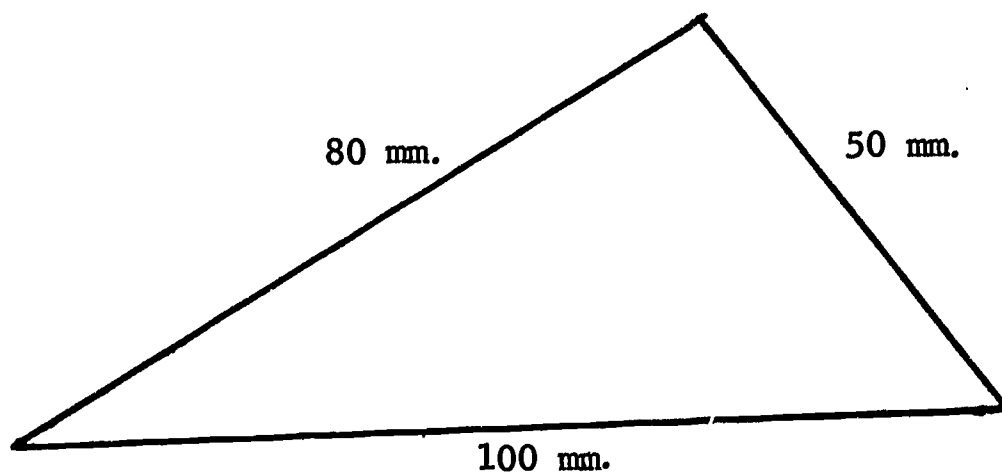
2. Did you get a set of three pairings which formed a set of equivalent fractions? _____ You should have.

3. Complete the table for these two similar triangles:

(Triangle 1)

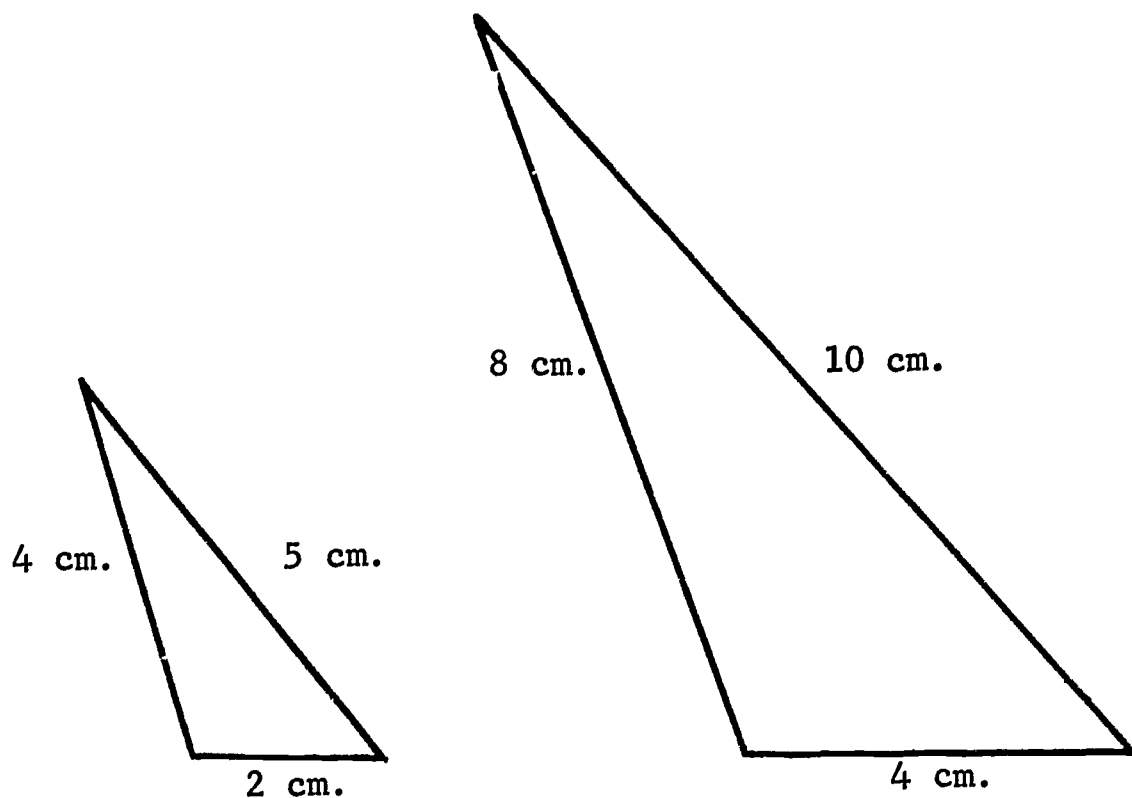


(Triangle 2)

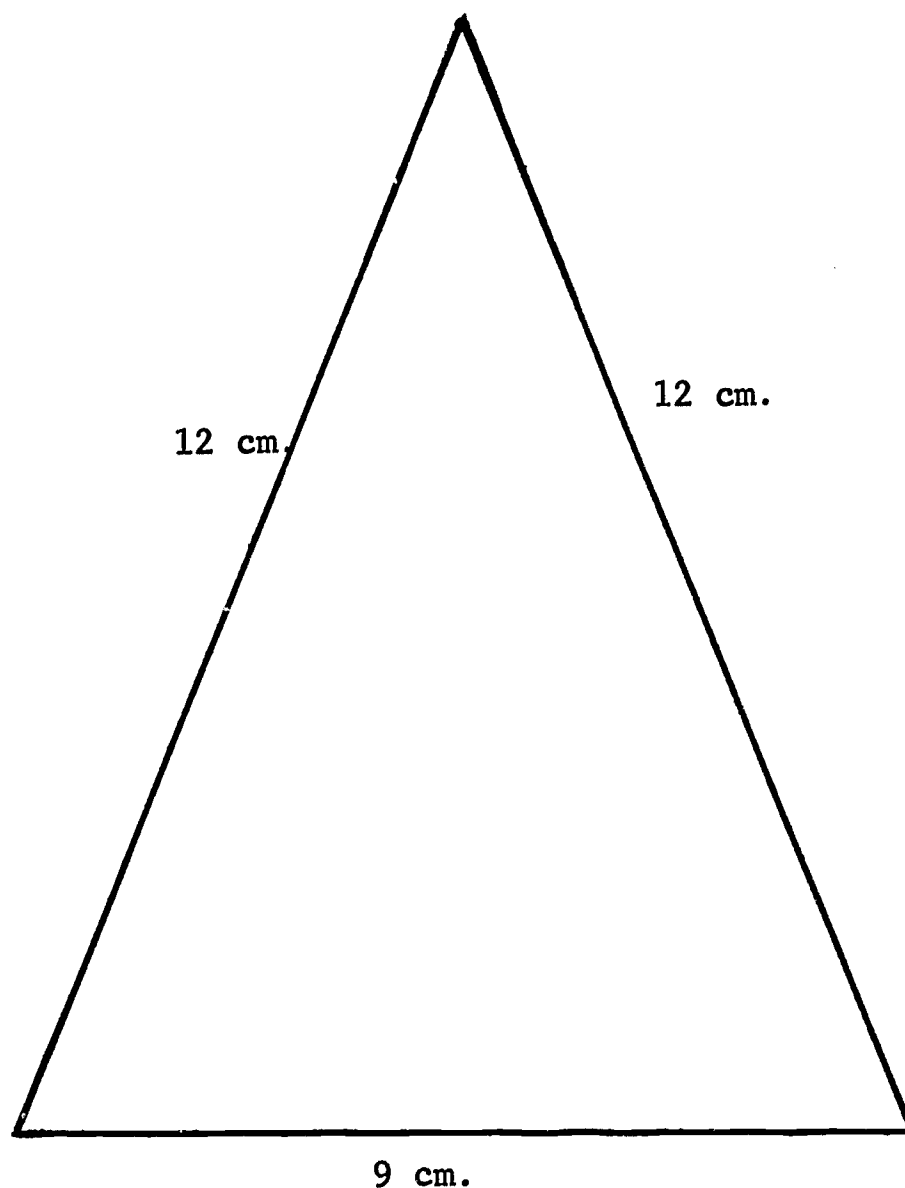
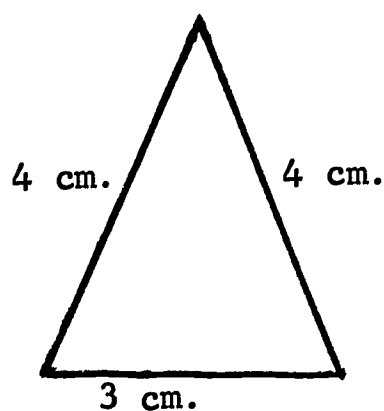


Lengths of sides of Triangle 1 (in mm)	Lengths of sides of Triangle 2 (in mm)	Correspondence or Pairing	Fractions
20	50	$20 \longleftrightarrow 50$	$\frac{20}{50}$
16	_____	_____	_____
10	_____	_____	_____
20	50	$20 \longleftrightarrow 50$	$\frac{20}{50}$
16	_____	_____	_____
10	_____	_____	_____
20	100	$20 \longleftrightarrow 100$	$\frac{20}{100}$
16	_____	_____	_____
10	_____	_____	_____
20	100	$20 \longleftrightarrow 100$	$\frac{20}{100}$
16	_____	_____	_____
10	_____	_____	_____
20	80	$20 \longleftrightarrow 80$	$\frac{20}{80}$
16	_____	_____	_____
10	_____	_____	_____
20	80	$20 \longleftrightarrow 80$	$\frac{20}{80}$
16	_____	_____	_____
10	_____	_____	_____

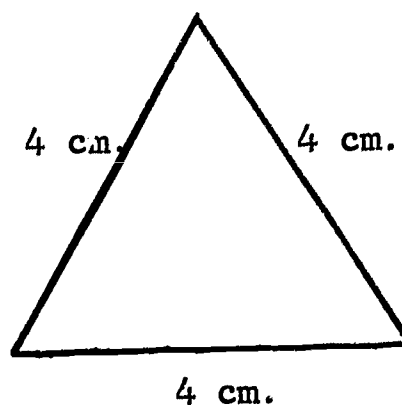
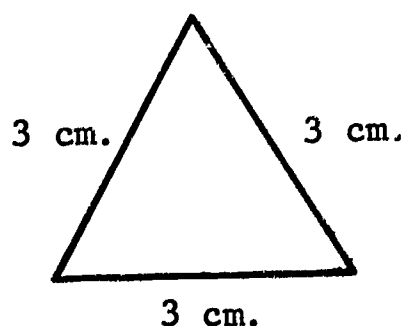
4. Did you get a set of three equivalent fractions in one of the pairings in problem 3? _____
5. Make a table like that of problems 1 and 3 for these two similar triangles:



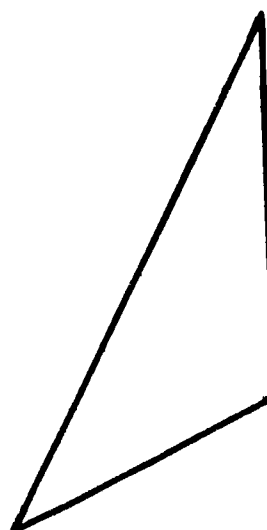
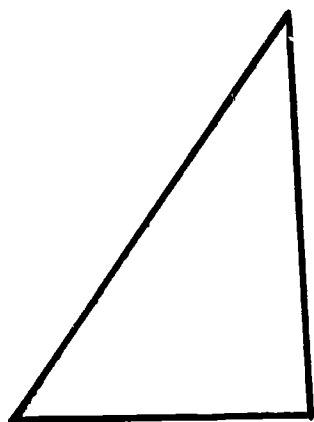
6. Did you get a set of three equivalent fractions in one of the pairings in problem 5? _____
7. Will you get at least one set of three equivalent fractions from the pairings of the measures of the lengths of the sides of two triangles when the two triangles are similar? _____
8. Make a table like that in problems 1, 3, and 5 for these two similar triangles: (There are only two different pairings for these two triangles.)



9. The two triangles pictured here are both equilateral. (An equilateral triangle is a triangle with all its sides the same length.) How many different pairings like those in problems 1, 3, 5, and 8 can be made between the lengths of the sides of these two triangles?



10. Are the fractions formed by the pairings in problem 9 equivalent? _____
11. Would you say that these two triangles have the same shape? _____
- Are these two triangles similar? _____

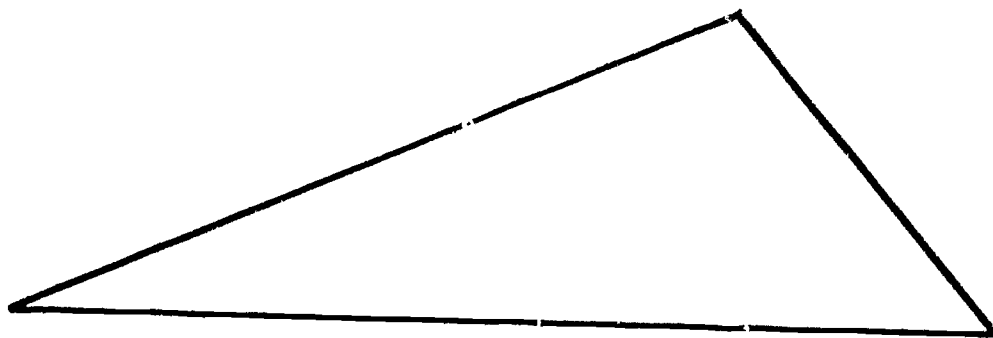


12. Use the metric scale on your ruler and find the length, to the nearest centimeter, of the sides of the two preceding triangles. Then set up the 6 correspondences between the lengths of the sides of the triangles--as was done in the previous activity. Write the sets of common fractions which go with these correspondences.

Lengths of sides of triangle 1. (in centimeters)	Lengths of sides of triangle 2. (in centimeters)	Pairings or Correspondence	Fractions

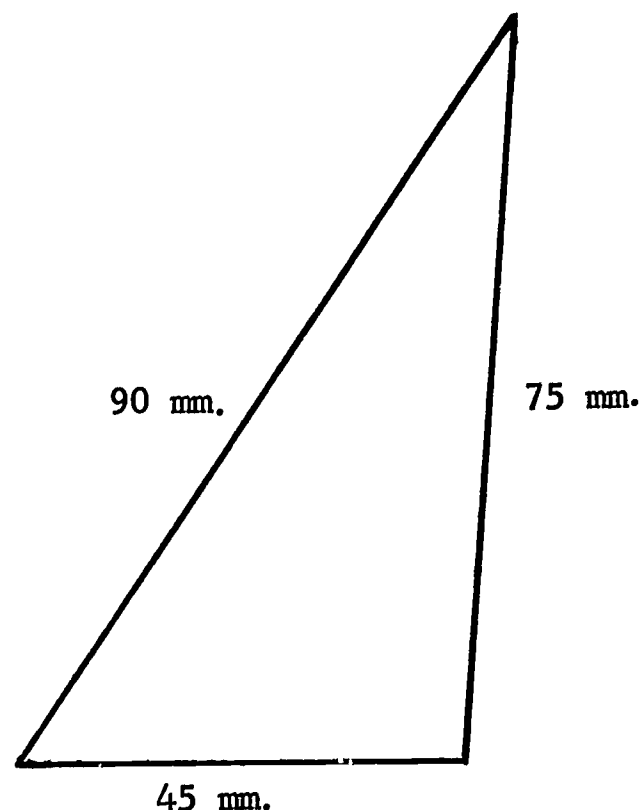
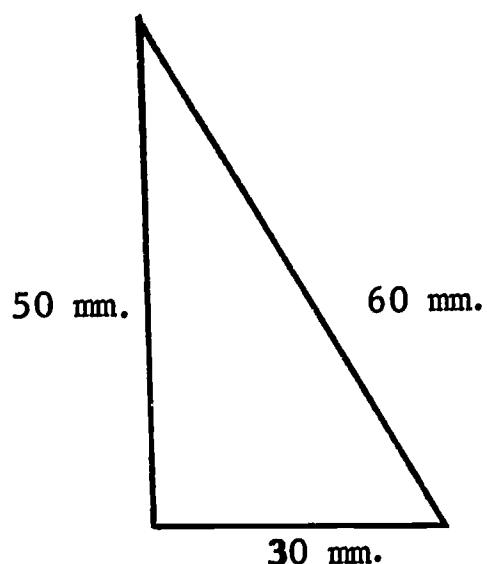
13. Of the 6 sets of common fractions, is one of the sets a group of three equivalent fractions? _____

14. Are the two triangles represented below similar? _____



15. With your ruler find the lengths of the sides of the two triangles in problem 14. Then set up the correspondences among the measures of the lengths of the sides of these two triangles and form the fractions. (These fractions are called ratios. Measure the lengths to the nearest centimeter.)

16. Now, do you think the two triangles of problem 14 are similar? _____
17. Are these two triangles similar? _____ Give two reasons for your answer.



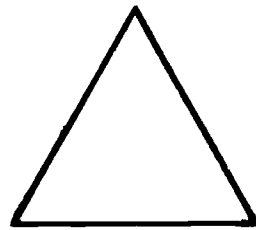
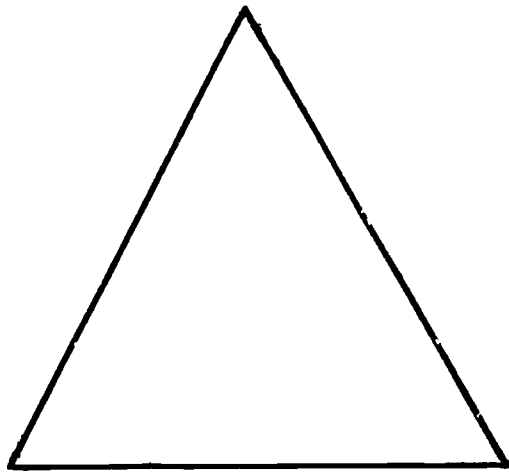
Do you see from problem 14 of the previous activities that it is not always possible to determine if two triangles are similar by "looking" at pictures of them. It is possible to determine if two triangles are similar provided the lengths of the sides of these triangles are known. Problems 12 and 13 of the set just completed give the information needed to do this.

Activities

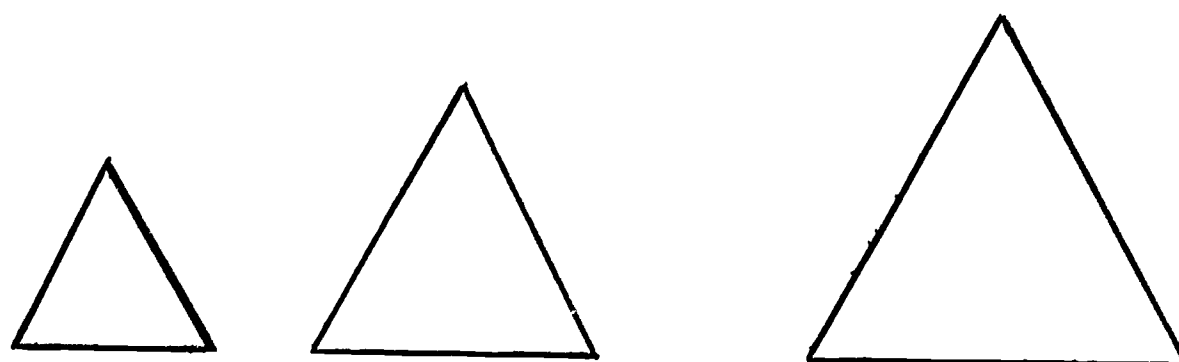
In each of the problems, 1 - 10, the lengths of the three sides of each of two triangles are given. Indicate which pairs are similar.

First TriangleSecond Triangle

- | | |
|--|-----------------------------|
| 1) 24 in., 27 in., and 9 in. | 40 in., 15 in., and 45 in. |
| 2) 32 cm., 36 cm., and 12 cm. | 40 cm., 45 cm., and 15 cm. |
| 3) 27 in., 15 in., and 13 in. | 54 in., 26 in., and 30 in. |
| 4) 38 mm., 57., and 76 mm. | 40 mm., 80 mm., and 60 mm. |
| 5) 8 ft., 9 ft., and 3 ft. | 63 ft., 21 ft., and 56 ft. |
| 6) 10 cm., 15 cm., and 20 cm. | 30 cm., 40 cm., and 50 cm. |
| 7) 24 yd., 26 yd., and 30 yd. | 36 yd., 39 yd., and 45 yd. |
| 8) 27 in., 30 in., and 15 in. | 45 in., 25 in., and 50 in. |
| 9) 33 in., 44 in., and 66 in. | 88 in., 99 in., and 121 in. |
| 10) 21 cm., 27 cm., and 12 cm. | 63 cm., 49 cm., and 28 cm. |
| 11) Pictured here are two equilateral triangles. Are they similar? _____ | |

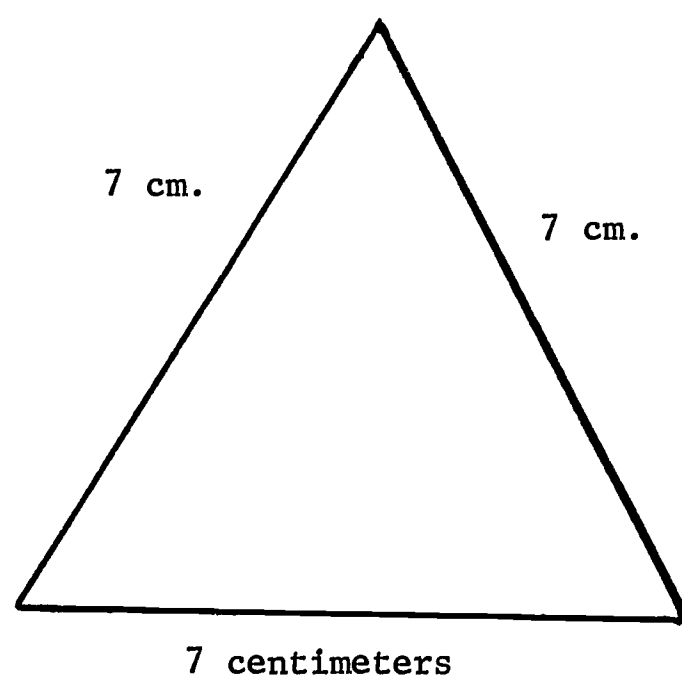
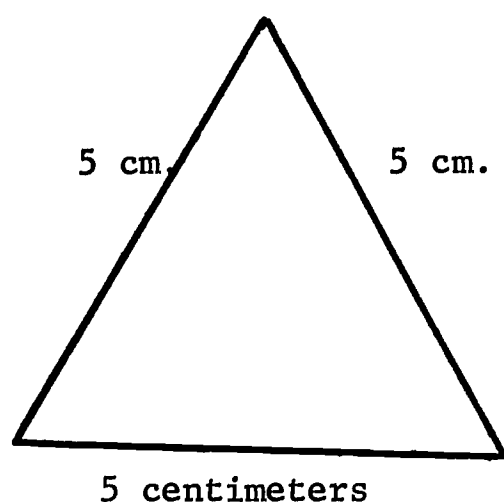


12. There are three equilateral triangles represented here. Are the three triangles similar? _____



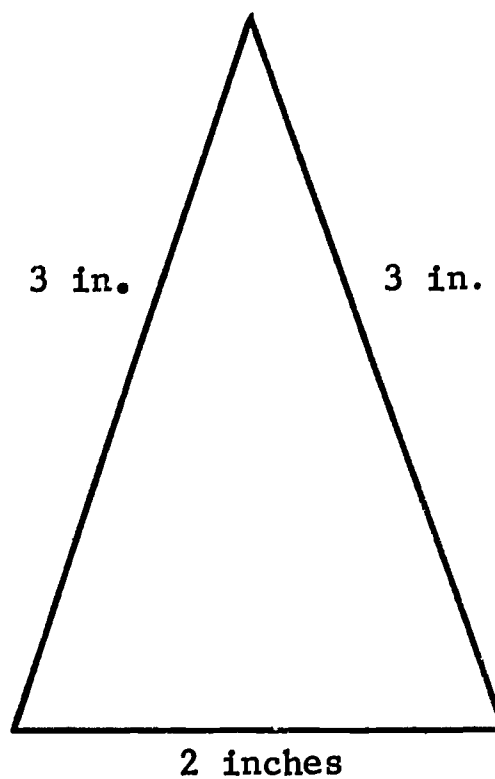
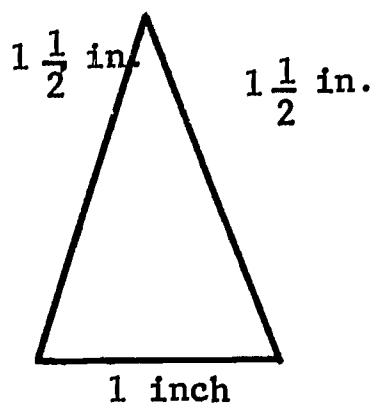
13. One triangle has 5 centimeters as the length of each of its sides.
A second triangle has 7 centimeters as the length of each of its sides.
Are the two triangles similar? _____

14. If you did not understand problem 13 or did not know the answer, use these two drawings to answer that same question here. _____



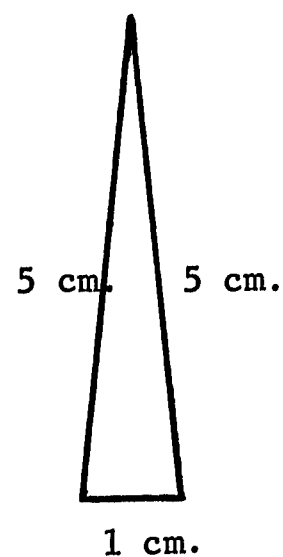
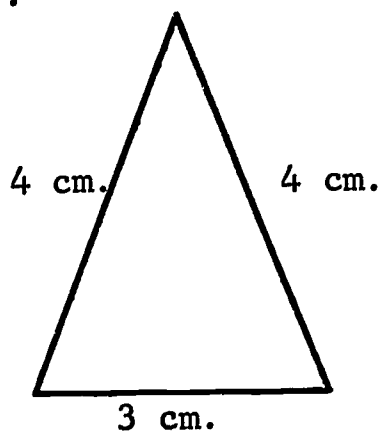
15. Are all equilateral triangles similar? _____

16. Represented here are two isosceles triangles. (Isosceles triangles have at least two sides with the same length.) Are these two triangles similar? _____

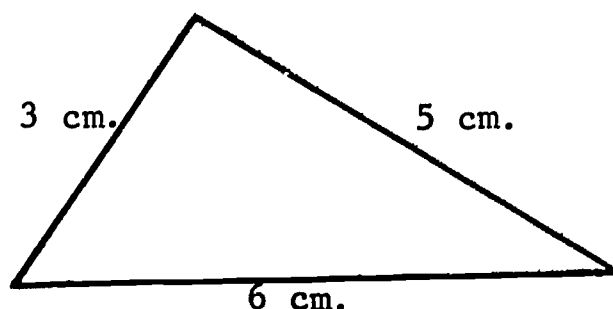


17. Are all isosceles triangles similar? _____

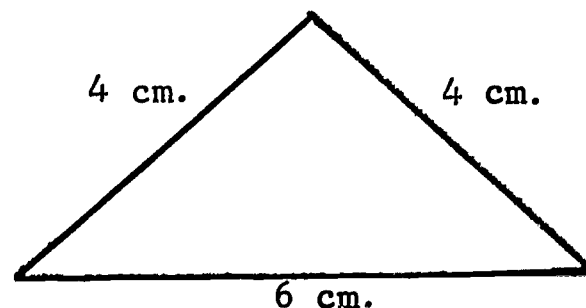
18. If your answer to problem 17 was yes, how do you explain that these two triangles represented here do not appear to be similar and yet they are isosceles?



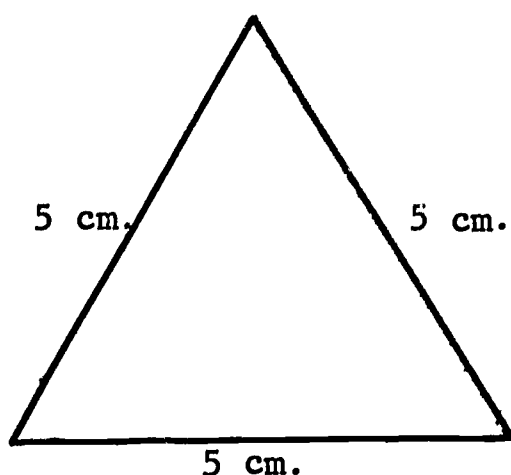
A triangle may be classified according to the lengths of its sides or according to measures of its angles. The following is a review of the classes or kinds of triangles:



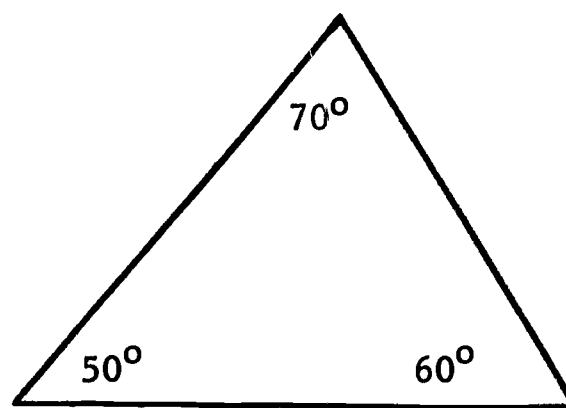
Scalene Triangle
(All sides have different lengths)



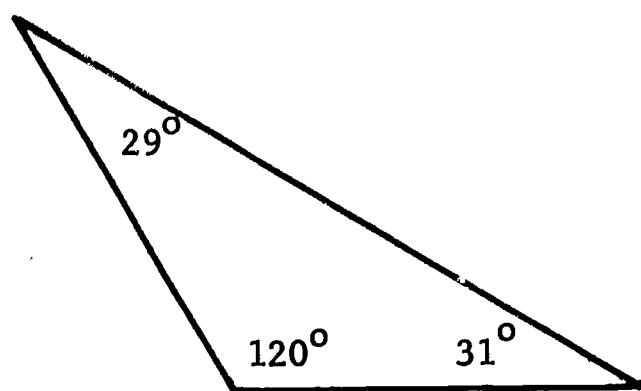
Isosceles Triangle
(At least 2 sides have same length)



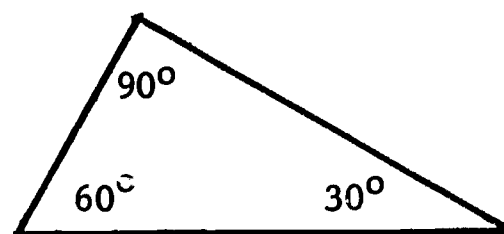
Equilateral Triangle
(All three sides have same length)



Acute Triangle
(Each angle has a measure less than 90°)



Obtuse Triangle
(Exactly one angle has measure greater than 90°)



Right Triangle
(Exactly one angle has a measure of 90°)

Study the following two examples:

Example 1: Determine if these two triangles are similar and then classify the two as to lengths of their sides and their angle measures. Triangle 1 has sides of lengths 15, 10, and 20 inches. Triangle 2 has sides of lengths 13, 19, and 22 inches.

Solution

Lengths of sides of triangle 1 (in inches)	Lengths of sides of triangle 2 (in inches)	Pairings or Correspondence	Fractions
15	13	$15 \longleftrightarrow 13$	$15/13$
10	19	$10 \longleftrightarrow 19$	$10/19$
20	22	$20 \longleftrightarrow 22$	$20/22=10/11$
15	13	$15 \longleftrightarrow 13$	$15/13$
10	22	$10 \longleftrightarrow 22$	$10/22=5/11$
20	19	$20 \longleftrightarrow 19$	$20/19$
15	19	$15 \longleftrightarrow 19$	$15/19$
10	13	$10 \longleftrightarrow 13$	$10/13$
20	22	$20 \longleftrightarrow 22$	$20/22=10/11$
15	19	$15 \longleftrightarrow 19$	$15/19$
10	22	$10 \longleftrightarrow 22$	$10/22=5/11$
20	13	$20 \longleftrightarrow 13$	$20/13$
15	22	$15 \longleftrightarrow 22$	$15/22$
10	19	$10 \longleftrightarrow 19$	$10/19$
20	13	$20 \longleftrightarrow 13$	$20/13$
15	22	$15 \longleftrightarrow 22$	$15/22$
10	13	$10 \longleftrightarrow 13$	$10/13$
20	19	$20 \longleftrightarrow 19$	$20/19$

Since none of the six sets of fractions is equivalent, then the two triangles are not similar. Both triangles are scalene. In triangle 1, since $20^2 > 10^2 + 15^2$, triangle 1 is obtuse. Since $22^2 < 19^2 + 13^2$ ($484 < 361 + 169$), triangle 2 is acute.

Example 2: Determine if these two triangles are similar and then classify the two as to lengths of their sides and their angle measures. Triangle 1 has sides of lengths 3, 4, and 5 inches. Triangle 2 has sides of lengths 9, 12, and 15 inches.

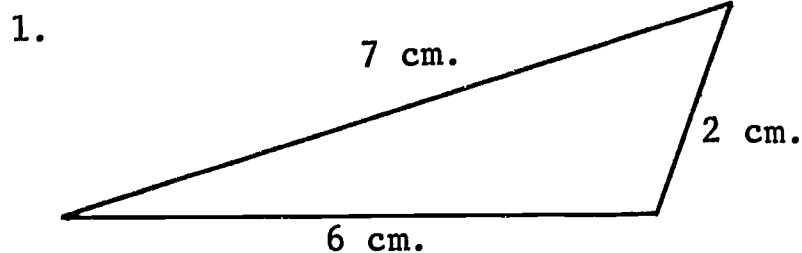
Solution

From the correspondence $3 \leftrightarrow 9$, and $4 \leftrightarrow 12$, and $5 \leftrightarrow 15$, the set of equivalent fractions $\left\{\frac{3}{9}, \frac{4}{12}, \frac{5}{15}\right\}$ is formed. This means the two triangles are similar. Both triangles are scalene. Since $5^2 = 3^2 + 4^2$, triangle 1 is a right triangle. Also, triangle 2 is a right triangle.

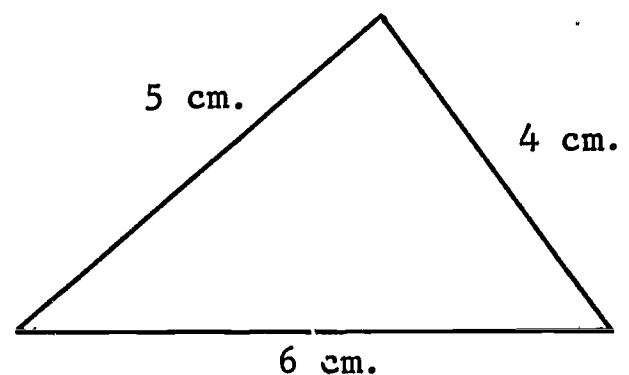
Activities

In each of the following pairs of triangles, determine if they are similar. Then classify each of the triangles according to the lengths of their sides and their angle measures.

Triangle 1



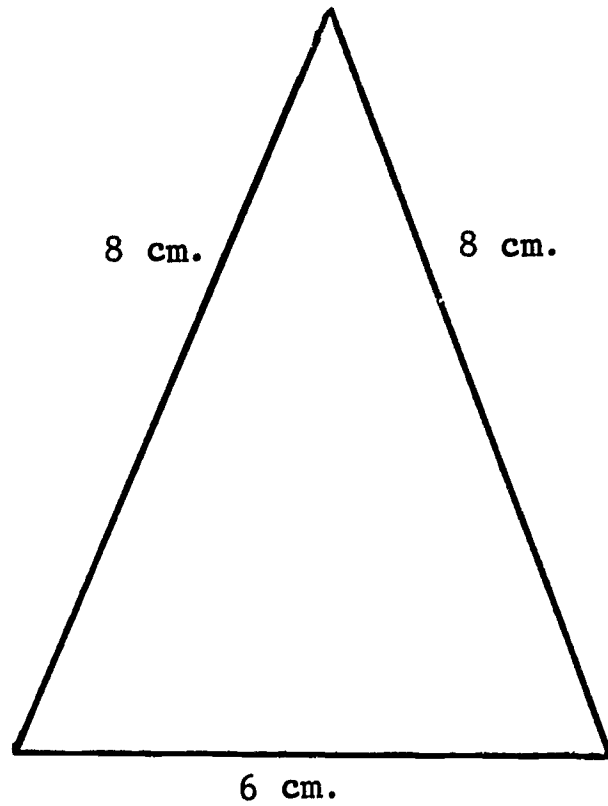
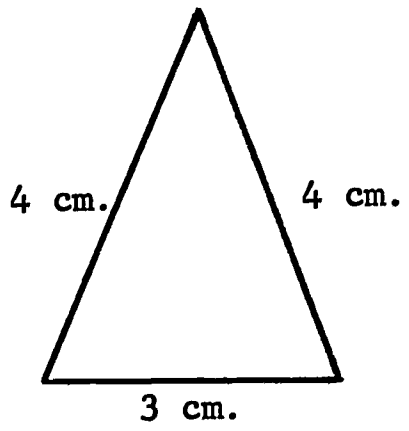
Triangle 2



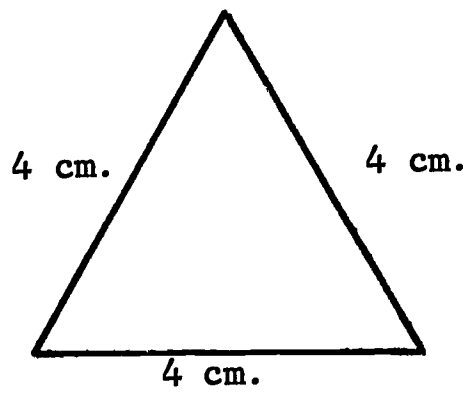
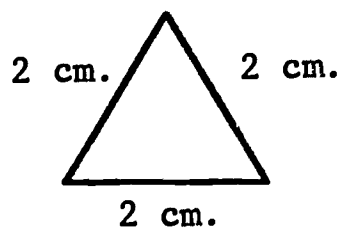
Triangle 1

Triangle 2

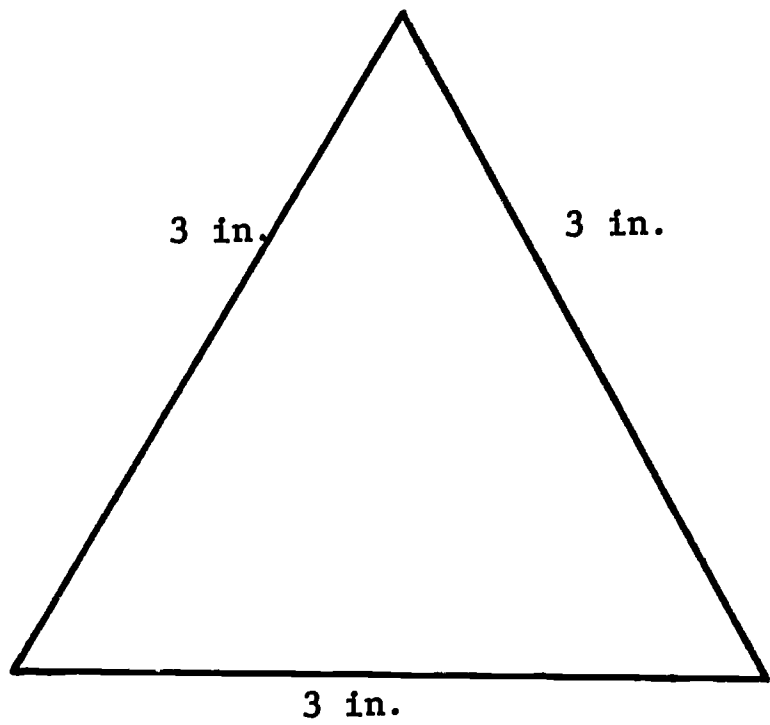
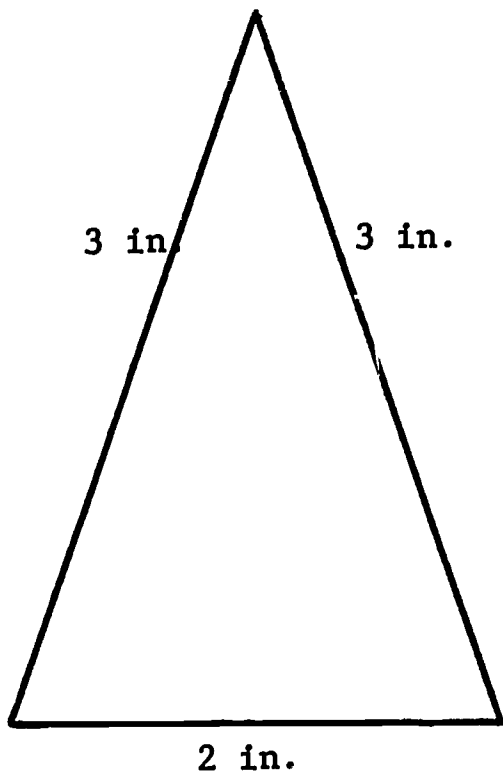
2.



3.

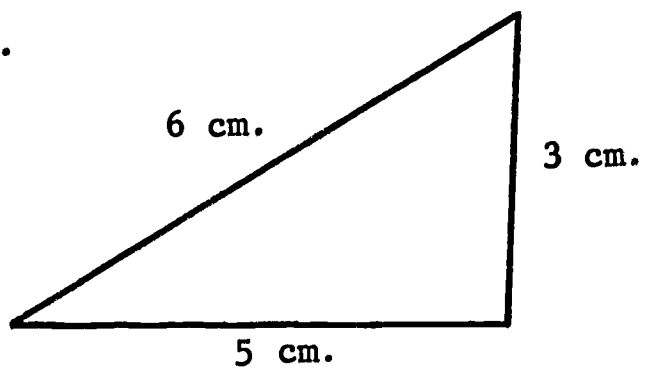


4.



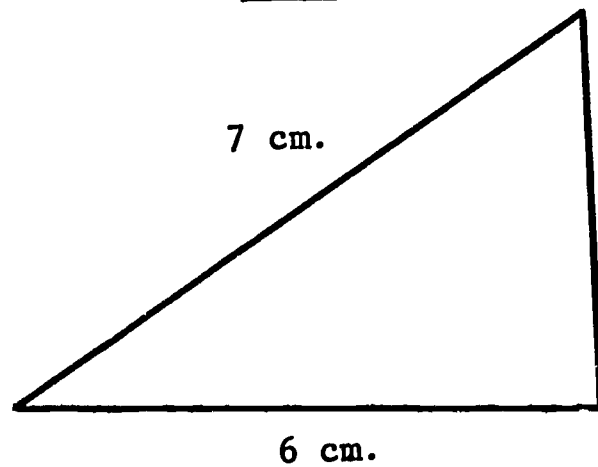
Triangle 1

5.

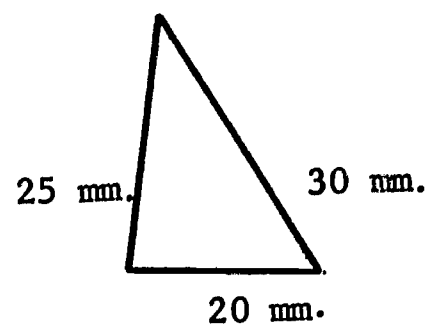
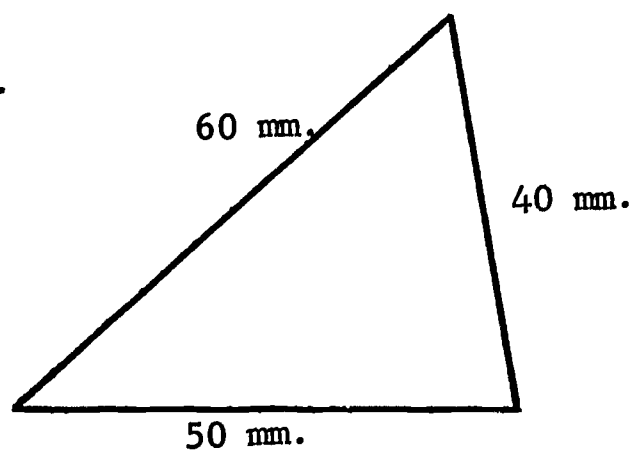
Triangle 2

7 cm.

4 cm.



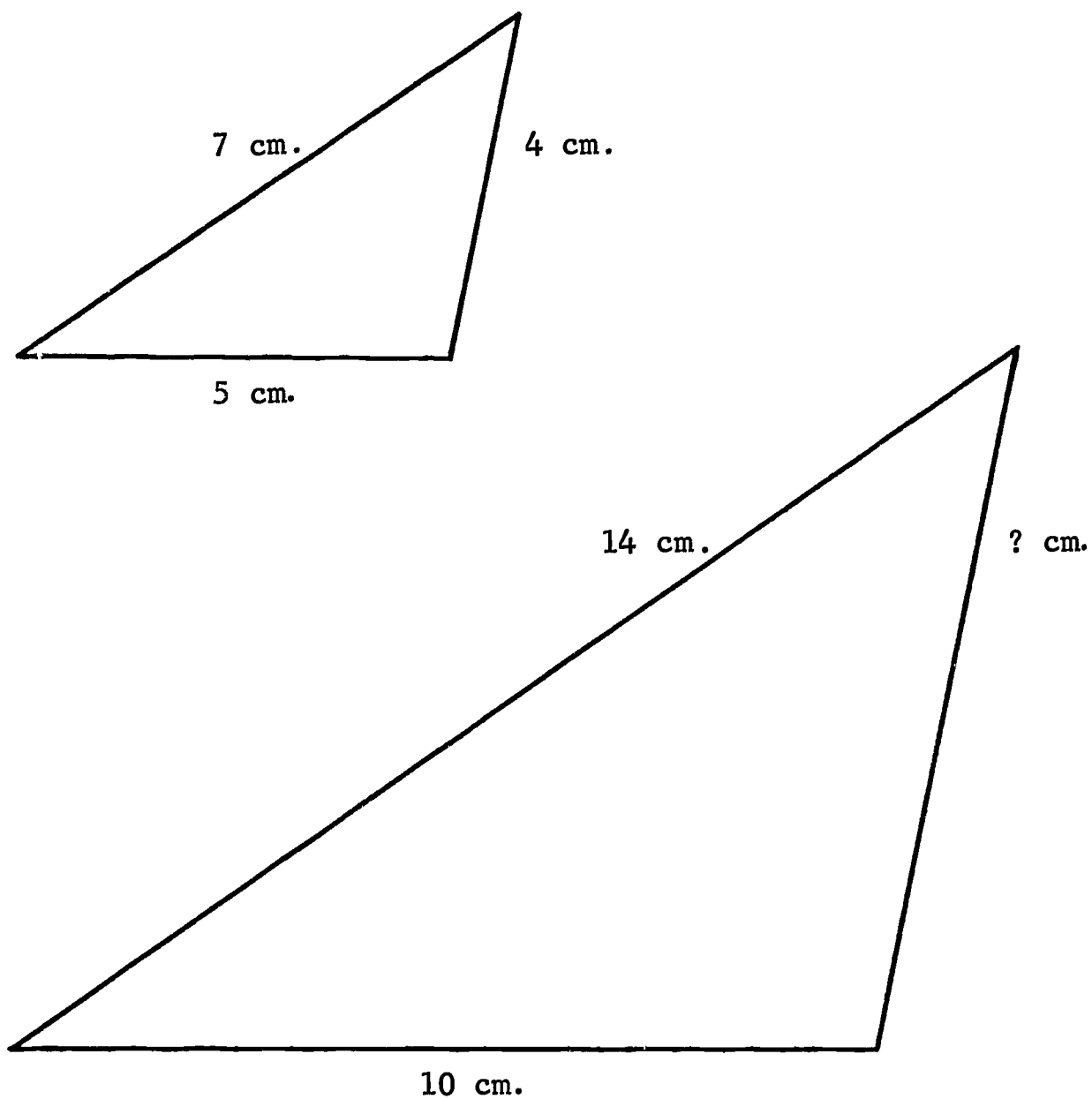
6.



- 7) 10, 15, and 20 inches
- 8) 4, 4, and 6 inches
- 9) 6, 8, and 12 inches
- 10) 15, 7, and 13 inches

- 30, 45, and 60 inches
- 10, 10, and 6 inches
- 5, 12, and 13 inches
- 15, 7, and 13 inches

The two triangles pictured below are similar. Without using your ruler, can you find the missing length of the side of the second triangle?



The length of the side with the question mark can be found by giving the missing denominator in the fraction below:

$$\frac{5}{10} = \frac{4}{?}$$

The length of the third side could also be found by supplying the missing denominator here:

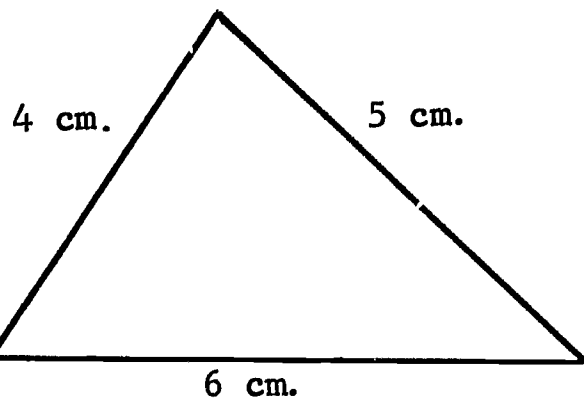
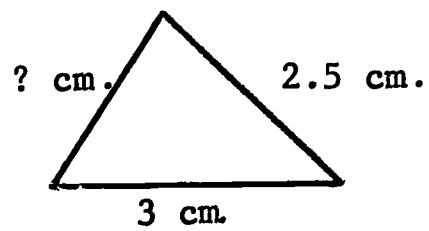
$$\frac{7}{14} = \frac{4}{?}$$

Activities

In each of these problems the two triangles are similar. Find missing length of the side of the triangle without using your ruler.

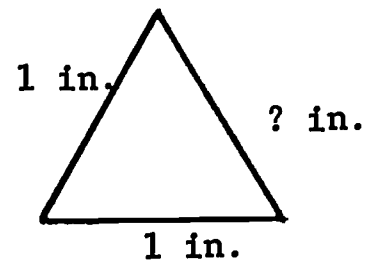
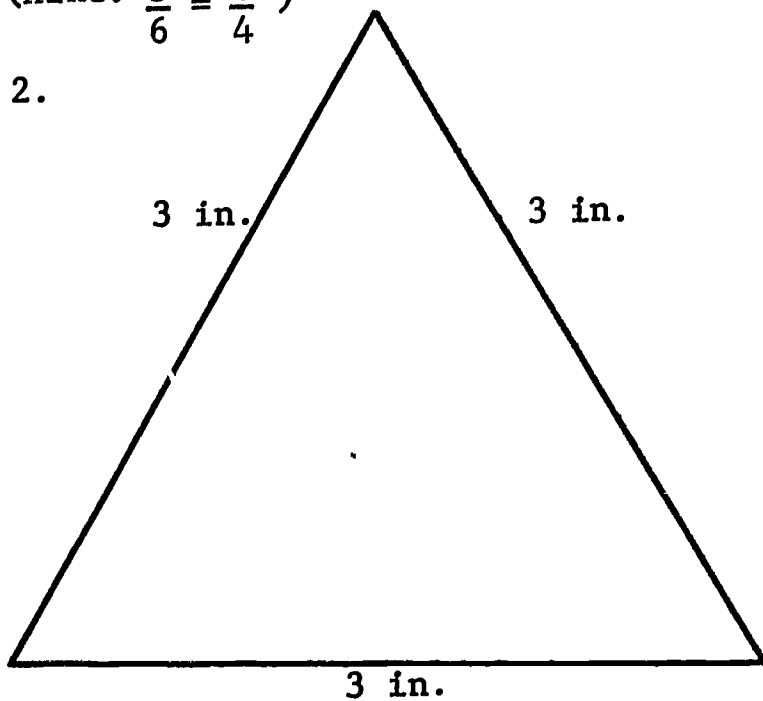
Triangle 1

1.

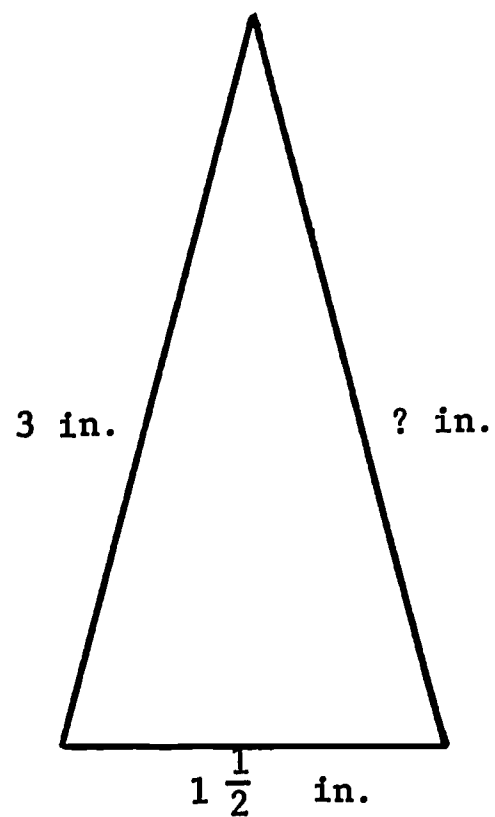
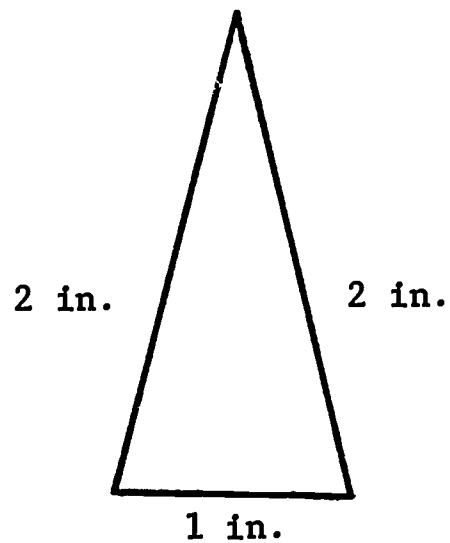
Triangle 2

(Hint: $\frac{3}{6} = \frac{?}{4}$)

2.



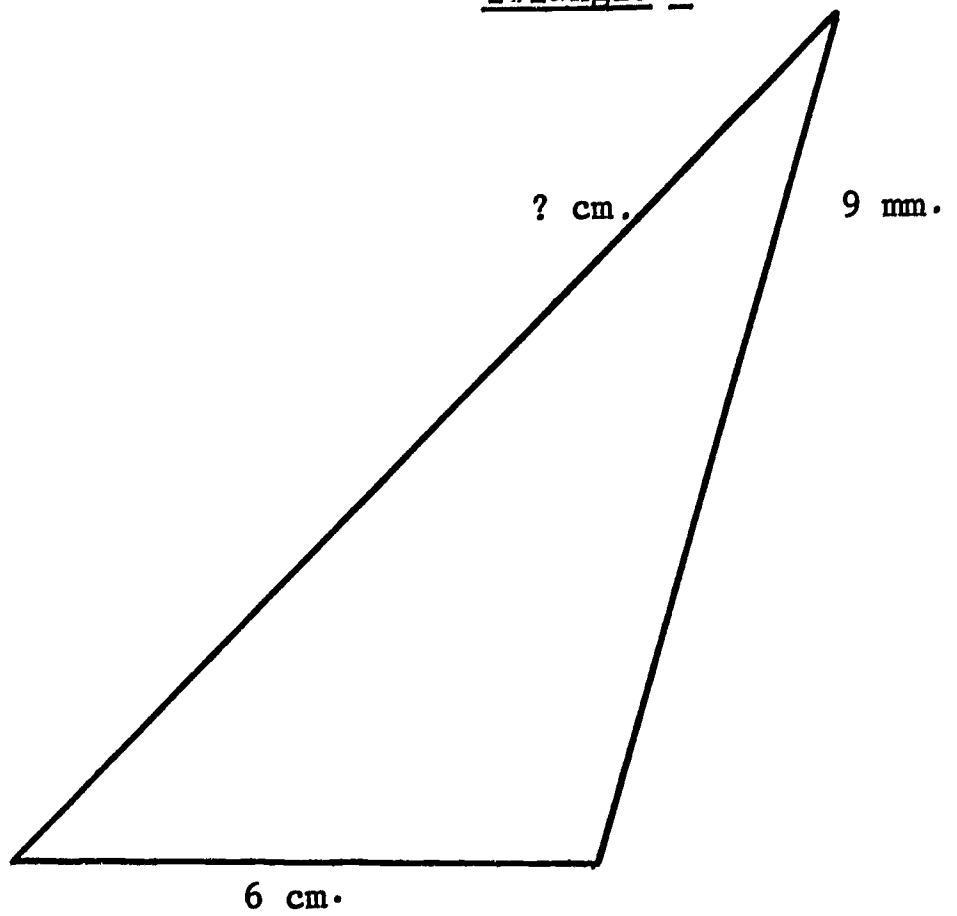
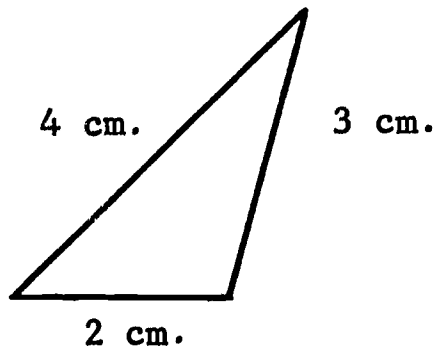
3.



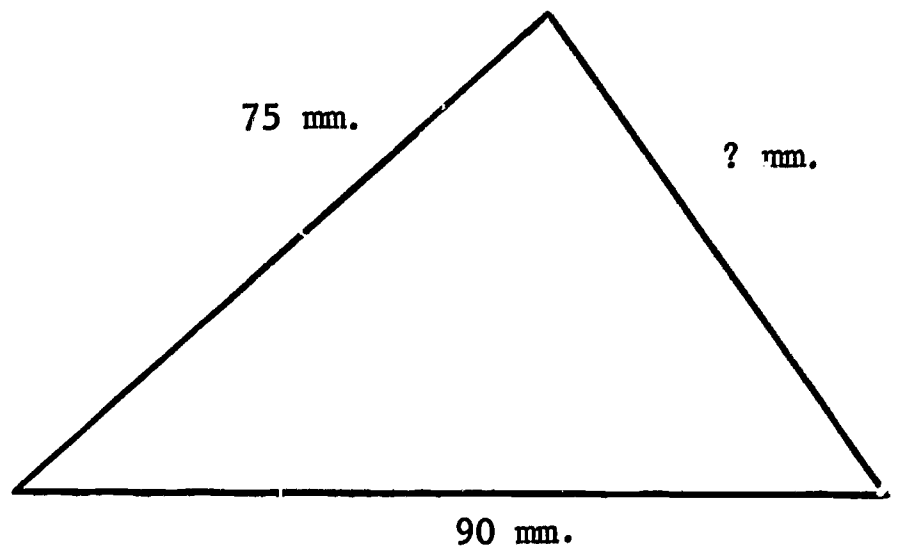
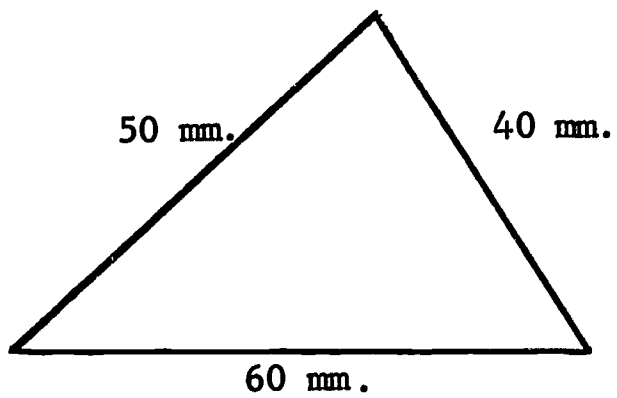
Triangle 1

Triangle 2

4.



5.



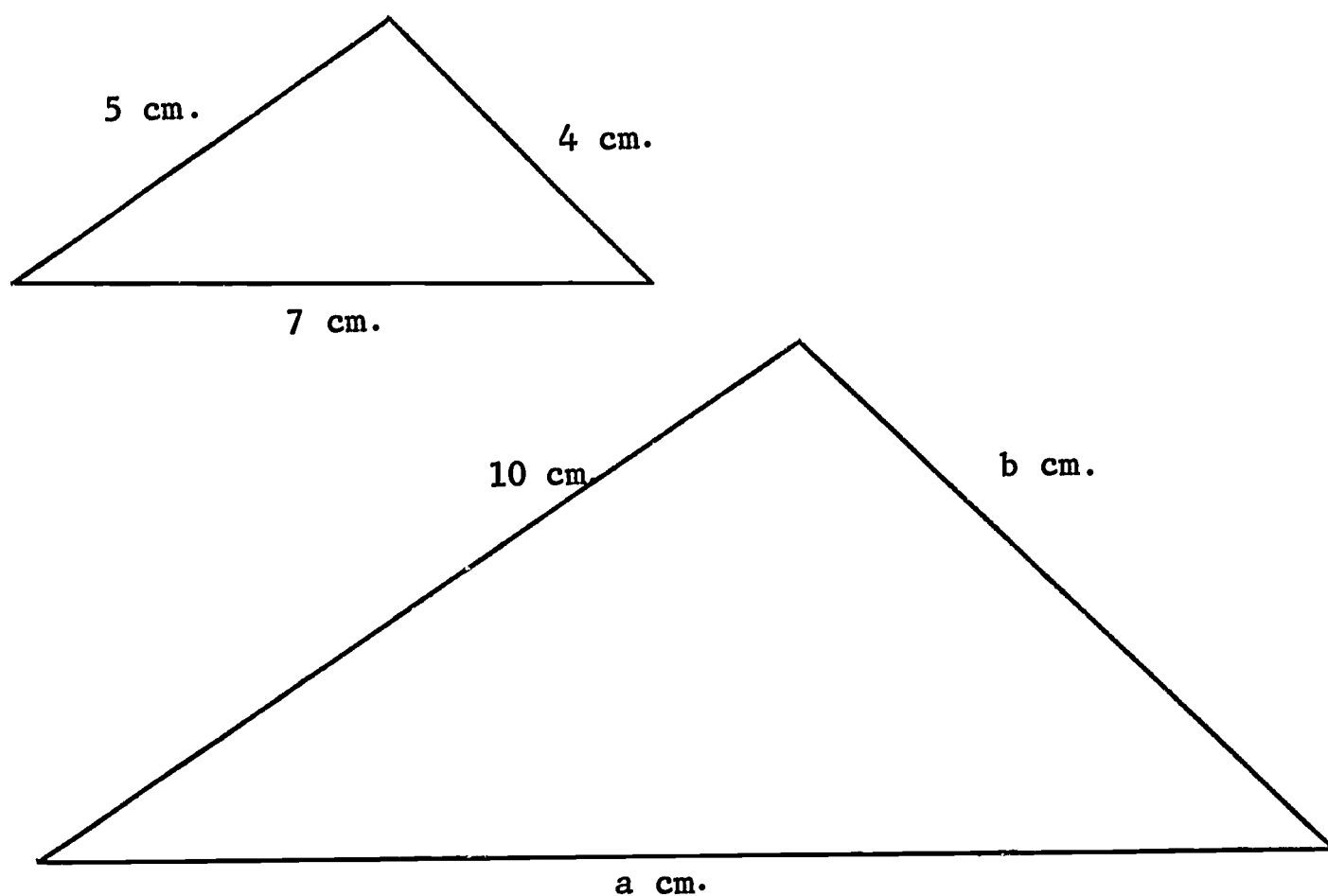
Triangle 1

- 6) 6, 13, and 11 inches
- 7) 6, 7, and 12 meters
- 8) 22, 18, and 24 inches
- 9) 8, 12, and 14 inches
- 10) 21, 21, and 27 inches

Triangle 2

- 18, 39, and _____ inches
- 24, 28, and _____ meters
- 11, 9, and _____ inches
- 20, 30, and _____ inches
- 28, 28, and _____ inches

The two triangles pictured below are similar. Without using your ruler, find the missing lengths of the two sides of the second triangle.



Solution: 1. $\frac{5}{10} = \frac{7}{a}$

2. $\frac{5}{10} = \frac{4}{b}$

$$\frac{1}{2} = \frac{7}{a}$$

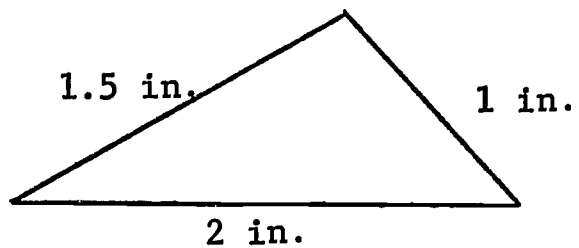
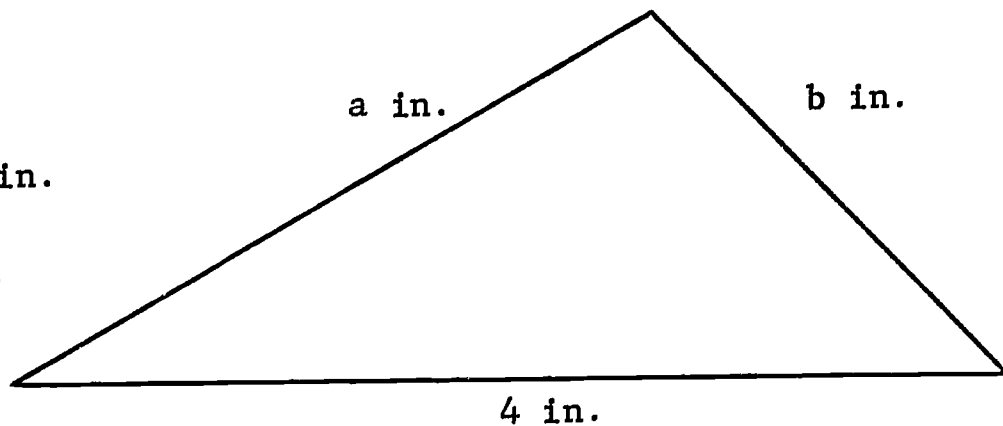
$$\frac{1}{2} = \frac{4}{b}$$

Activities

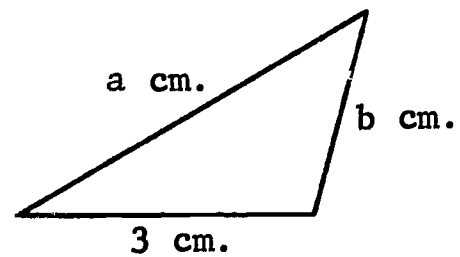
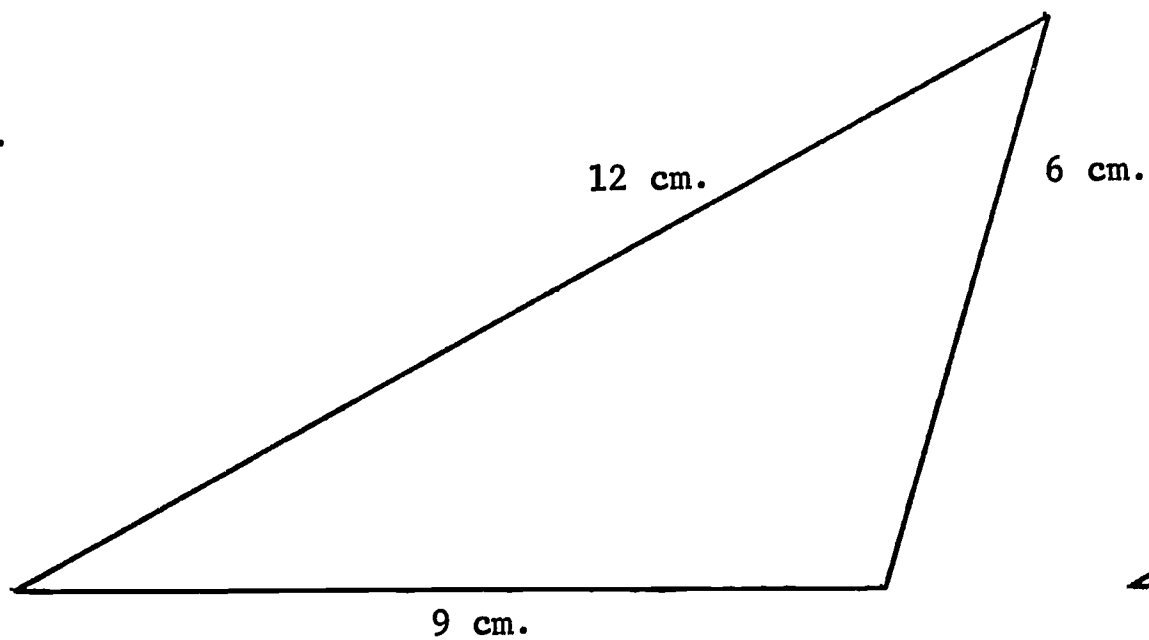
In each of these problems the two triangles are similar. Find the missing lengths of the two sides of the triangle without using your ruler.

Triangle 1

1.

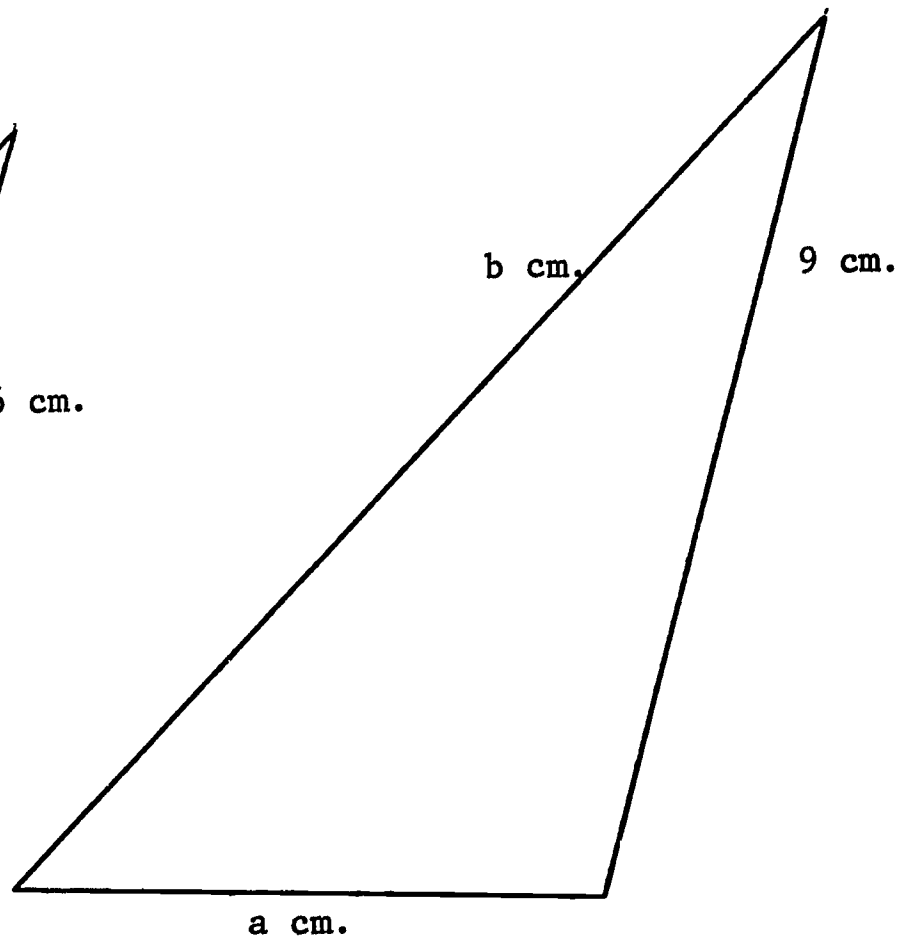
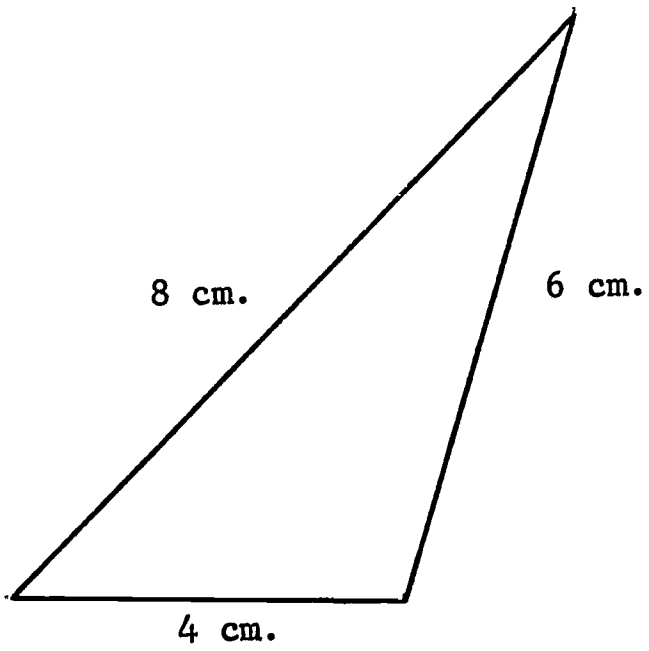
Triangle 2

2.

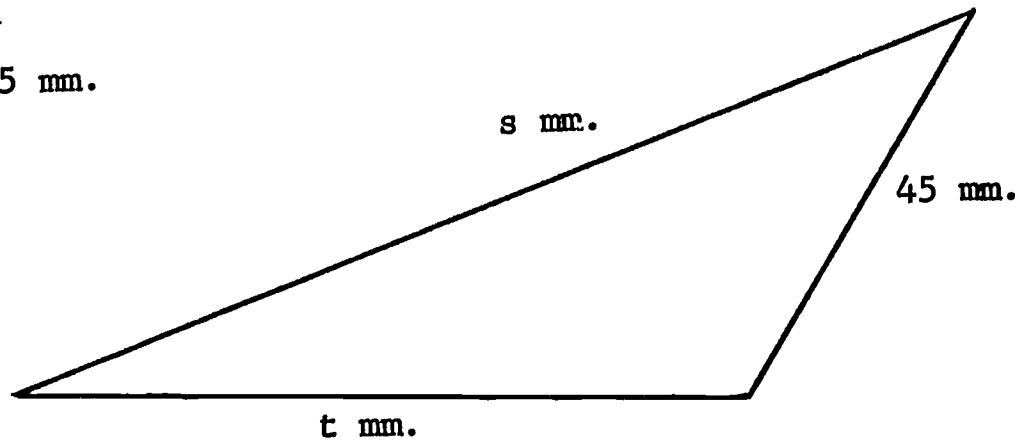
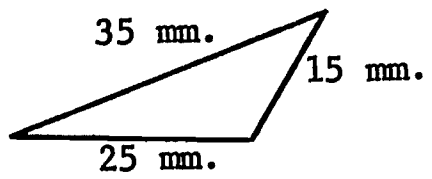


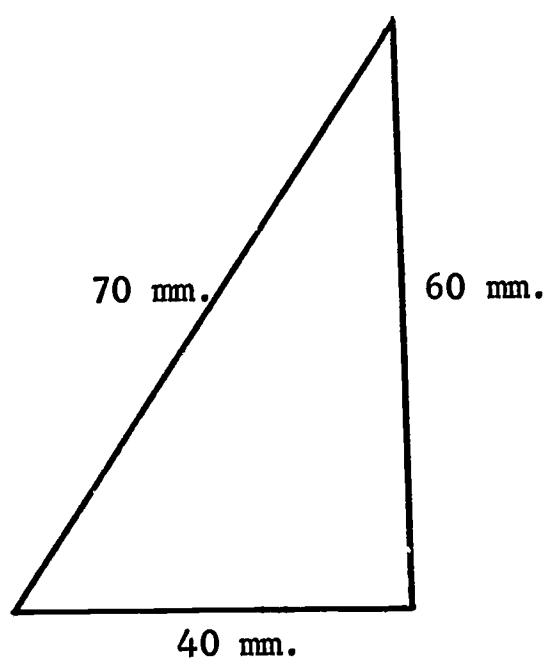
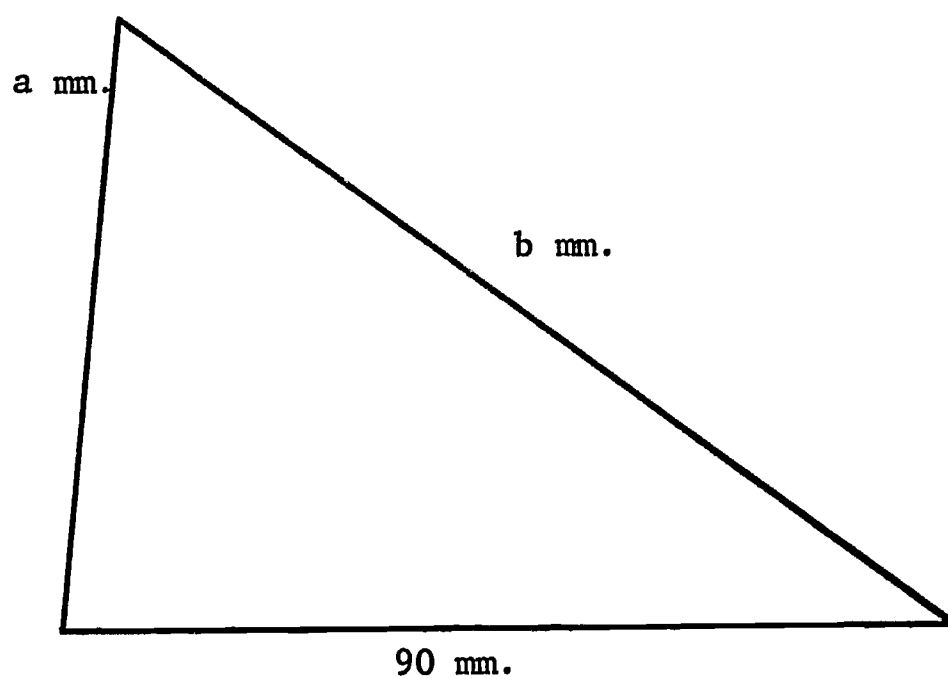
Triangle 1Triangle 2

3.



4.



Triangle 1Triangle 2

- 6) 12, 16, and 20 inches
- 7) 15, 10, and 23 inches
- 8) 6, 7, and 8 inches
- 9) 10, 20, and 25 centimeters
- 10) 12, 9, and 18 inches

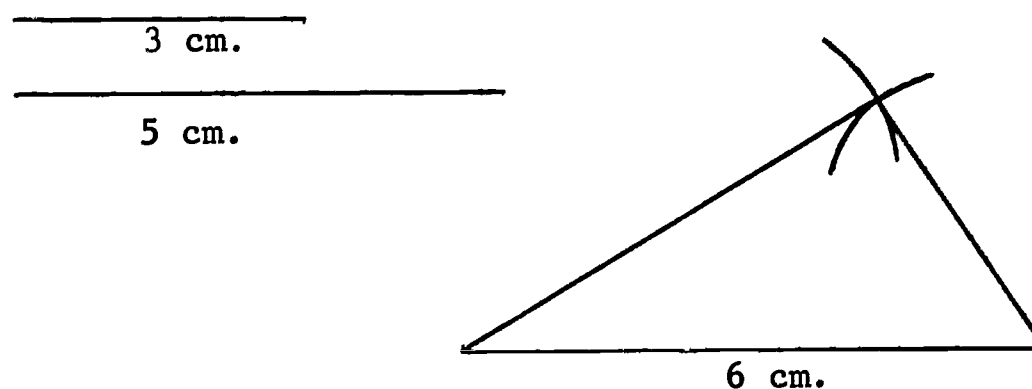
- 3, _____, and _____ inches
- 30, _____, and _____ inches
- 18, _____, and _____ inches
- _____, _____, and 50 centimeters
- _____, 27, and _____ inches

It is necessary for you to learn to "construct," with a compass and ruler, a triangle.

Example: "Construct" a triangle with sides of length 3, 5, and 6 centimeters.

Solution

With your ruler draw line segments of lengths 3, 5, and 6 centimeters.



Use the line segments of lengths 3 and 5 centimeters to set the radius of your compass. With the radius of your compass set at 3 cm. and the point of the compass on one end of the segment of length 6 cm., draw an arc as in the preceding figure. With your compass set at a radius of 5 cm. and the point on the other end of the 6 cm. segment draw an arc intersecting (meeting) the first arc. Draw line segments from the ends of the 6 cm. segment to the point of intersection of the two arcs. (See drawing.)

Activities

In each of the problems 1 - 5 "construct" a triangle which has sides of the given lengths:

- 1) 3, 4, and 5 centimeters
- 2) 3, 4, and 4 centimeters
- 3) 5, 5, and 5 centimeters
- 4) 6, 9, and 4 centimeters
- 5) 35, 40, and 45 millimeters

In each of the problems 6 - 8 "construct" a triangle whose sides have the same lengths as the segments shown:

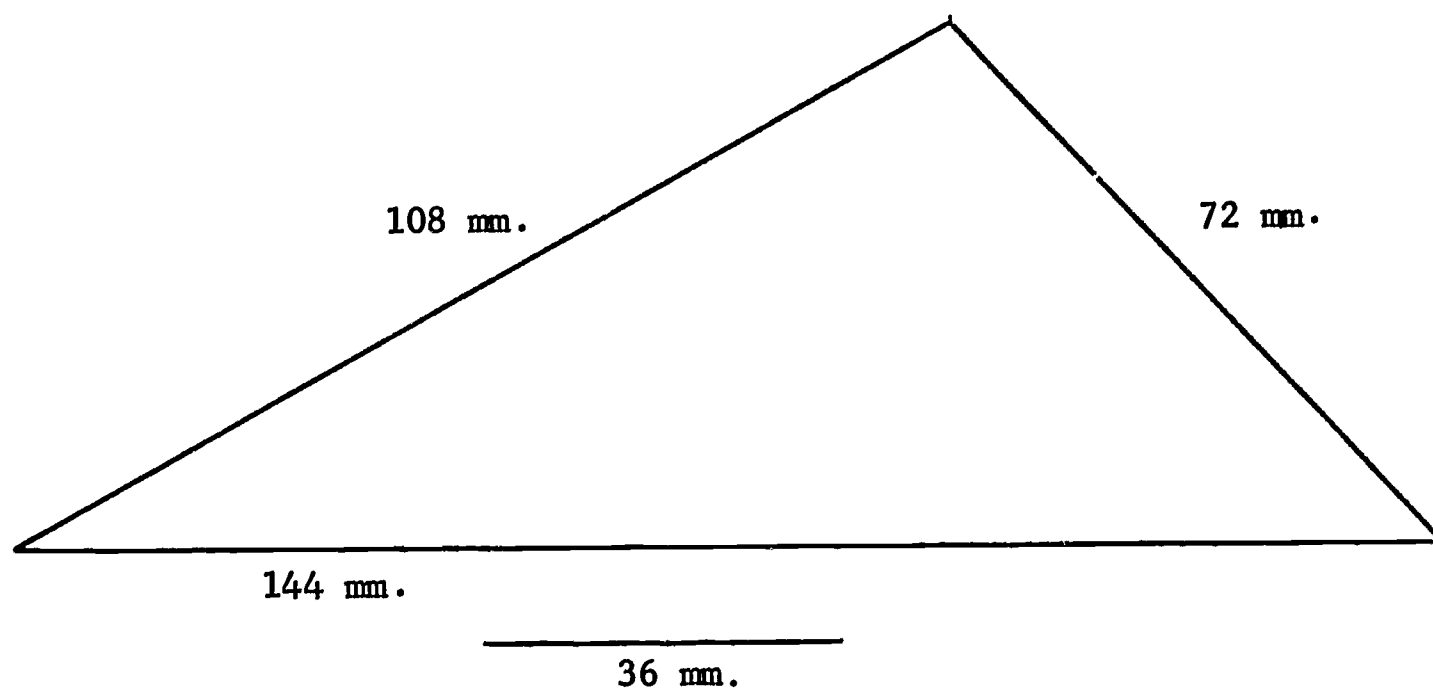
6. _____

7. _____

8. _____

9. Can you "construct" a triangle with sides of lengths 1, 2, 3 inches? Try it.
10. Can you "construct" a triangle with sides of lengths 2, 3, and 6 centimeters? Why or why not?

Example: "Construct" all the triangles possible which have one side of length 36 millimeters and are similar to the following triangle:



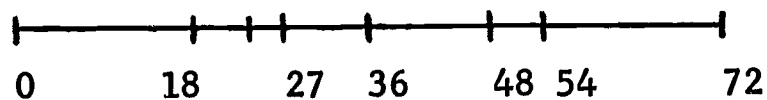
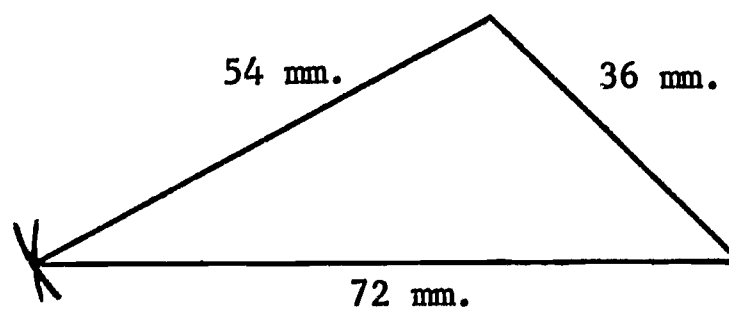
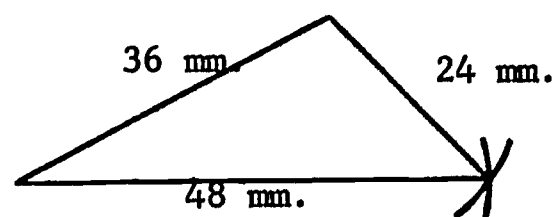
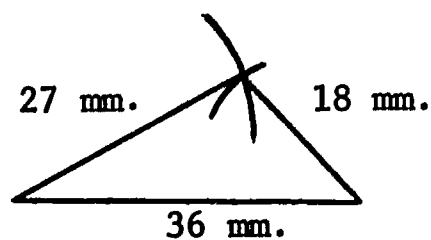
Pairings (Lengths of sides in mm.)

1. 144 \longleftrightarrow 36
 108 \longleftrightarrow ?
 72 \longleftrightarrow ?

2. 144 \longleftrightarrow ?
 108 \longleftrightarrow 36
 72 \longleftrightarrow ?

3. 144 \longleftrightarrow ?
 108 \longleftrightarrow ?
 72 \longleftrightarrow 36

Constructions



(Scale in mm.)

Do you remember that two similar geometric figures are either the same size or one is an enlargement of the other? In the first part of this example the larger triangle is "four times larger" than the triangle you are to construct because 144 is four times 36. ($144 = 4 \times 36$) The lengths of the other two sides of the larger triangle must also be four times the length of the two sides of the smaller triangle.

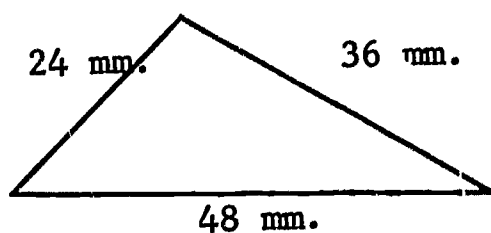
$$108 = 4 \times ?$$

$$72 = 4 \times ?$$

Activities

In each of the problems 1 - 5 "construct" all the triangles possible which have a side with the given length and are similar to the given triangle.

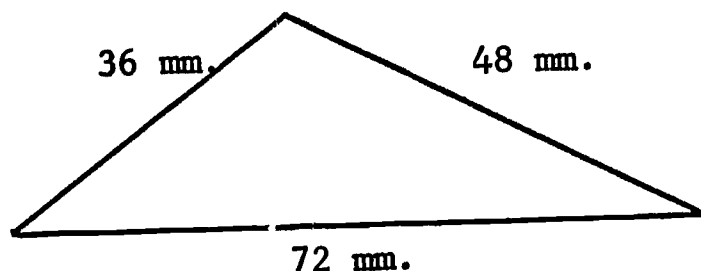
1.



24 mm.

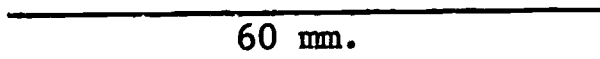
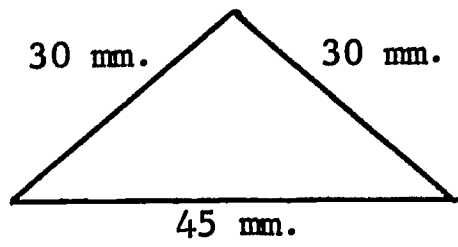
Hint: $24 \leftrightarrow 24$ $36 \leftrightarrow ?$ $48 \leftrightarrow ?$ $24 \leftrightarrow ?$ $36 \leftrightarrow 24$ $48 \leftrightarrow ?$ $24 \leftrightarrow ?$ $36 \leftrightarrow ?$ $48 \leftrightarrow 24$

2

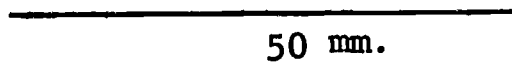
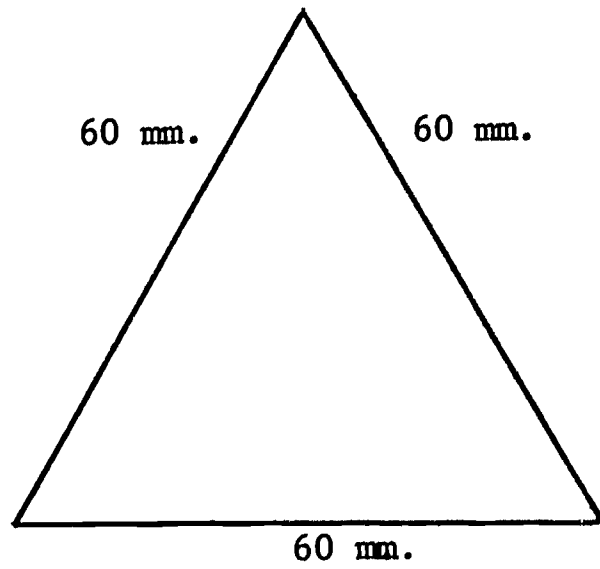


24 mm.

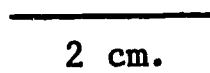
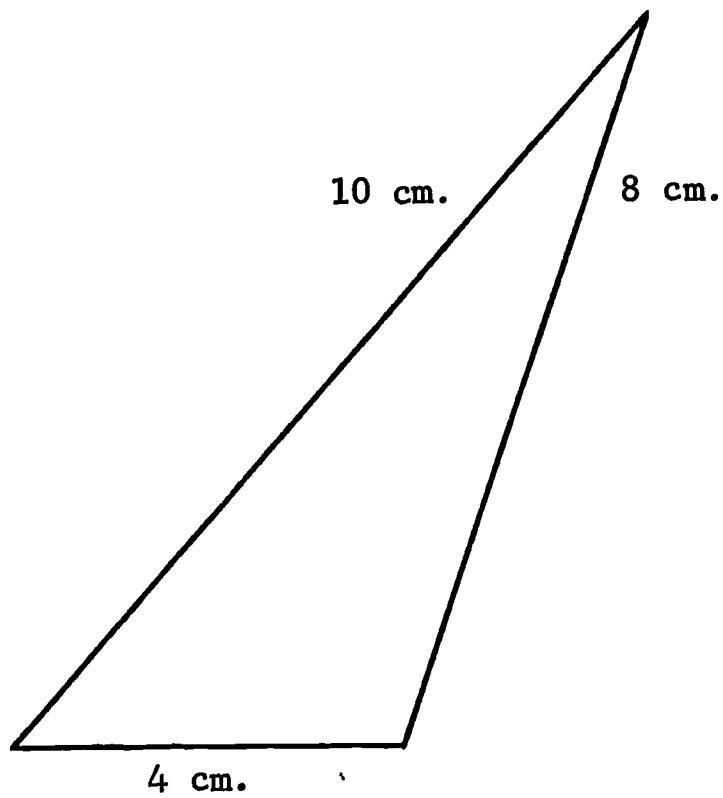
3.



4.



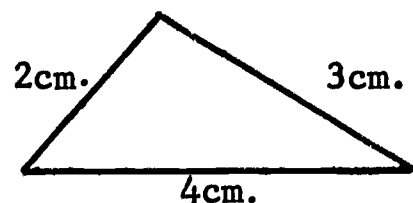
5.



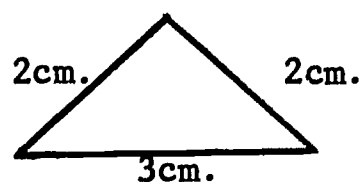
In each of the problems 6 - 10 how many triangles with the given requirements may be constructed similar to the given triangle? Do not construct them.

Given Triangles

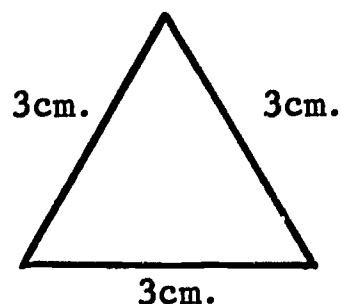
6.



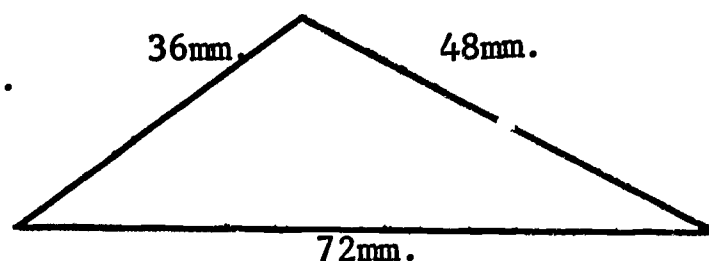
7.



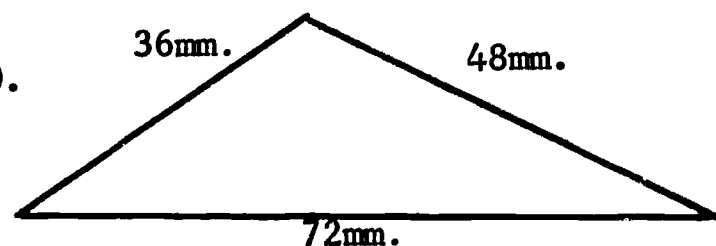
8.



9.



10.



Given Requirements

One side of length 1 centimeter.

1 cm.

At least one side of length 6 centimeters.

6 cm.

At least one side of length 5 centimeters.

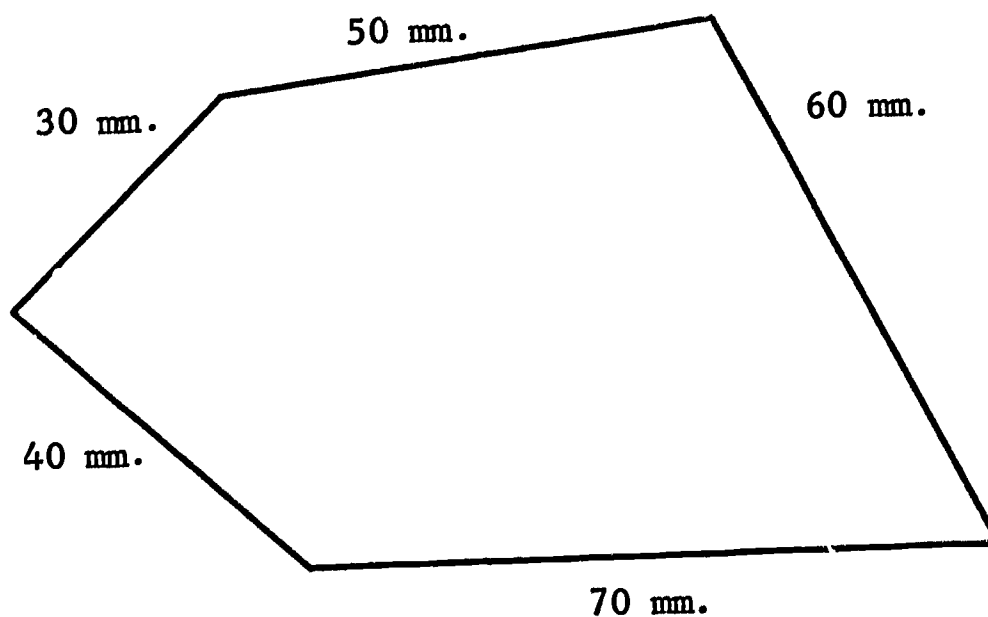
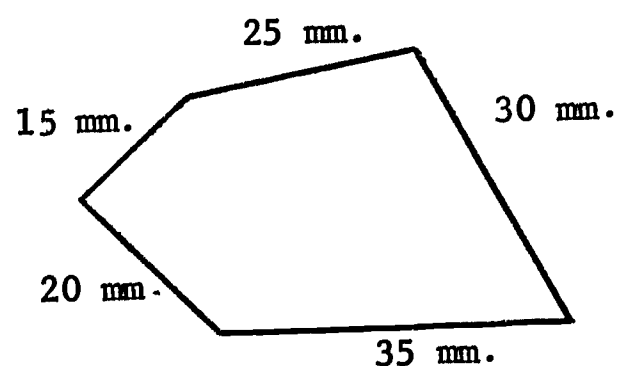
5 cm.

Two sides, one 12 millimeters in length and the other 24 millimeters in length.

Two sides, one 6 millimeters in length and the other 24 millimeters in length.

Similar Polygons

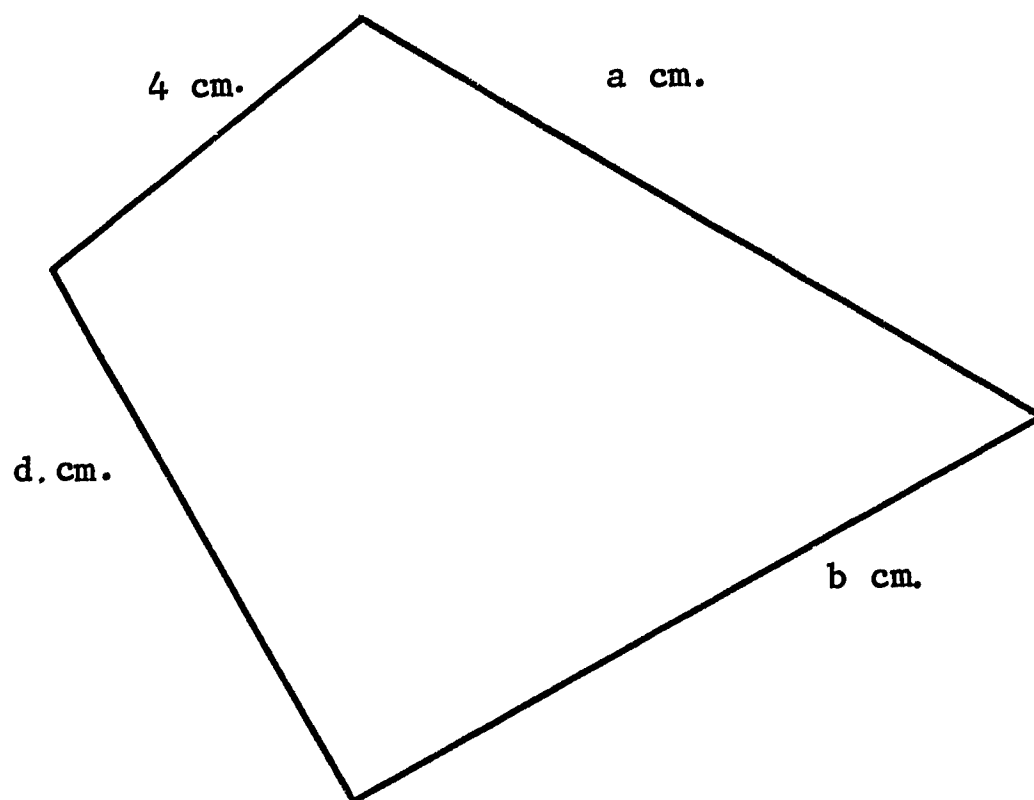
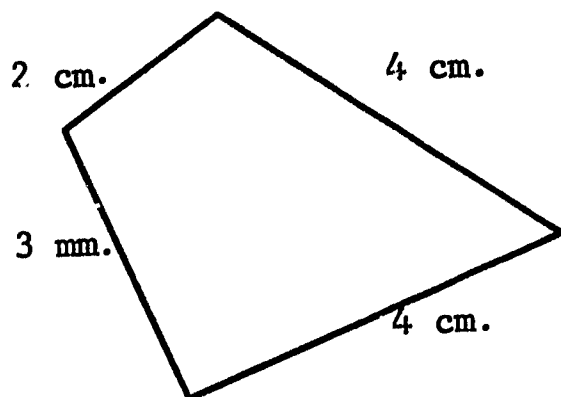
The two pentagons pictured here are similar. Notice that one of the correspondences (pairings) forms a set of equivalent fractions.



$$\left\{ \frac{35}{70}, \frac{20}{40}, \frac{15}{30}, \frac{25}{50}, \frac{30}{60} \right\}$$

Example: Find the missing lengths of the sides of the second quadrilateral.

The two quadrilaterals are similar.



$$\frac{2}{4} = \frac{4}{a}; a = \underline{\hspace{2cm}}$$

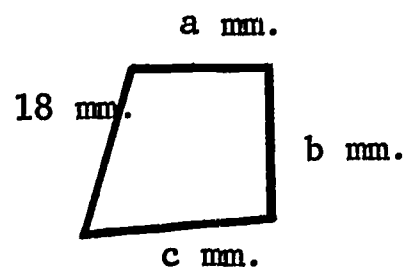
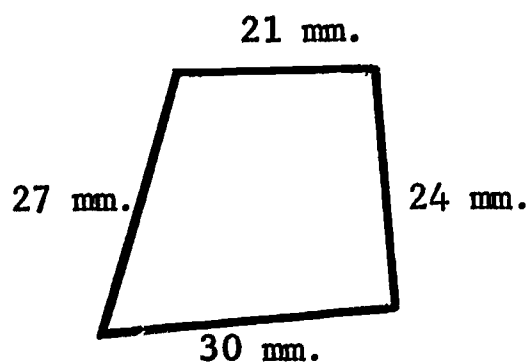
$$\frac{2}{4} = \frac{4}{b}; b = \underline{\hspace{2cm}}$$

$$\frac{2}{4} = \frac{3}{d}; d = \underline{\hspace{2cm}}$$

Activities

In each of the following problems find the missing lengths, without using your ruler, of the sides of the second polygon. The two polygons in each problem are similar.

1.

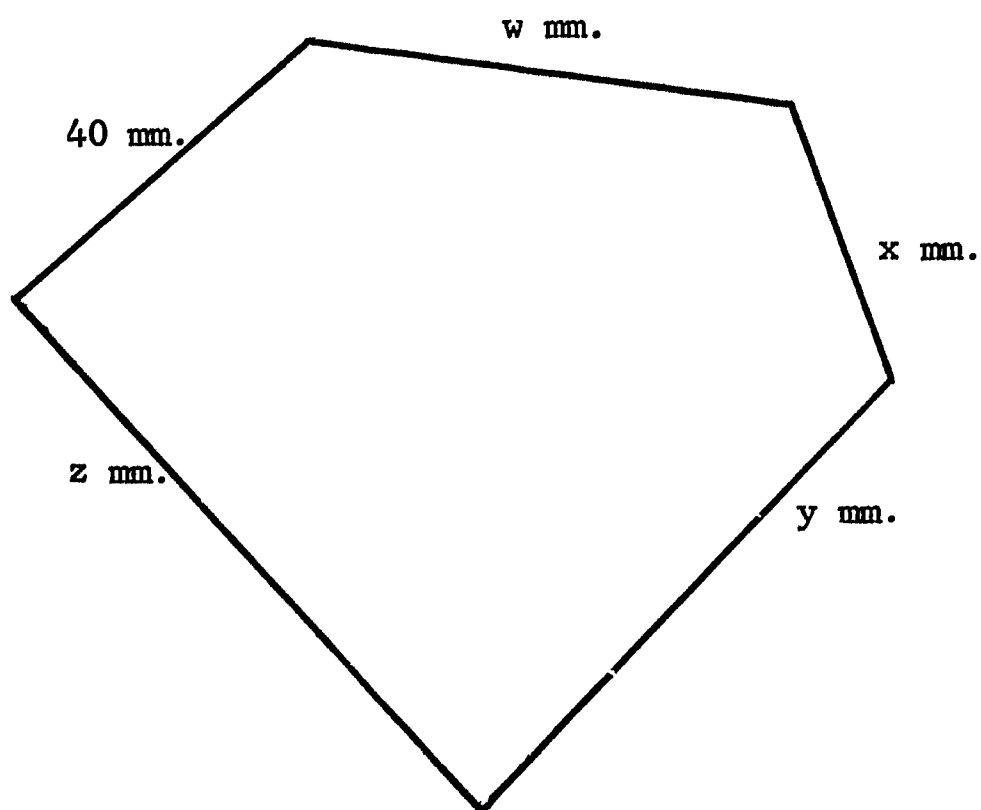
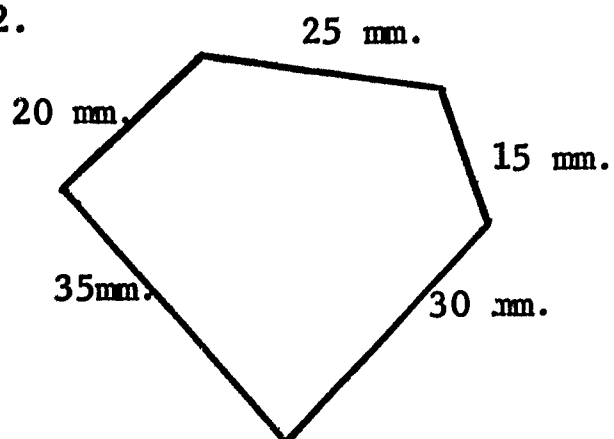


a = _____

b = _____

c = _____

2.



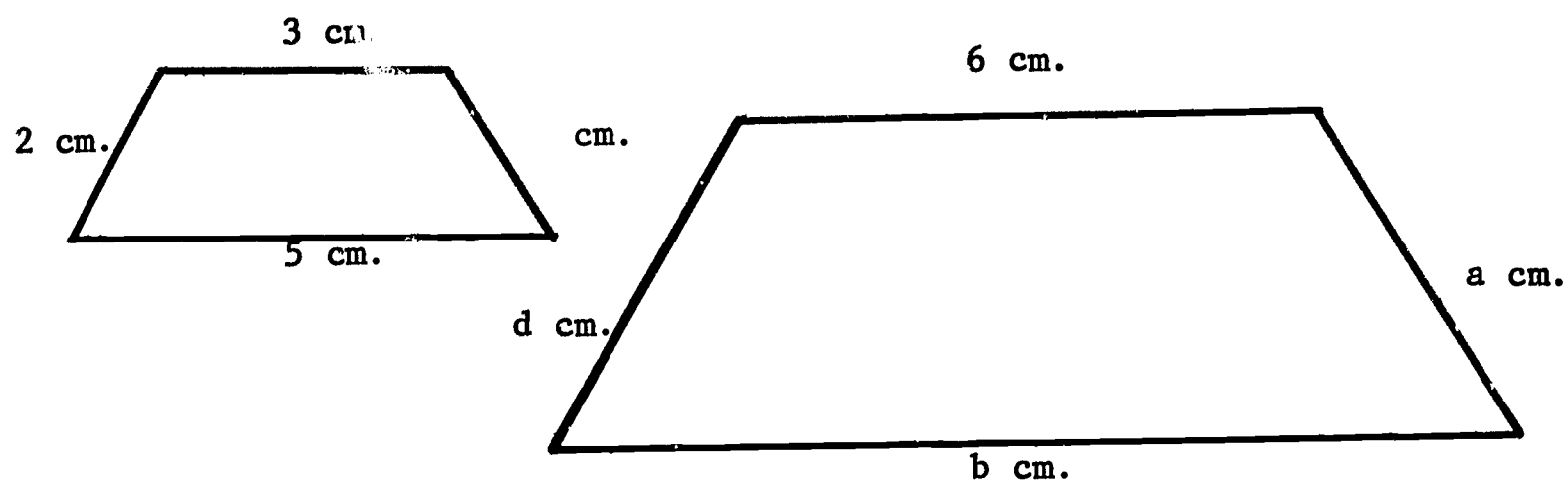
w = _____

x = _____

y = _____

z = _____

3.

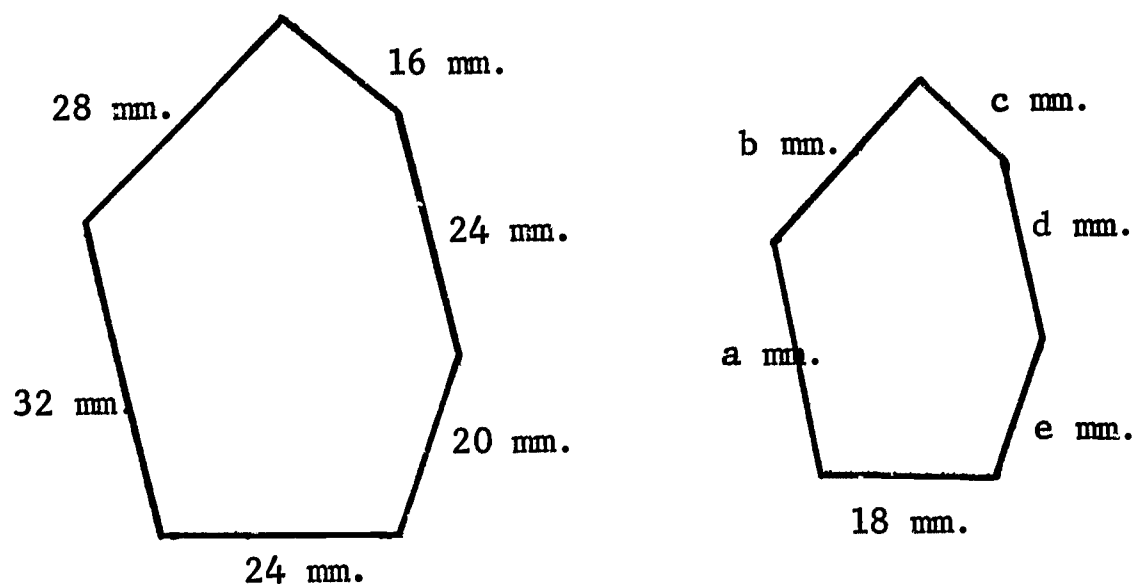


a = _____

b = _____

d = _____

4.



a = _____

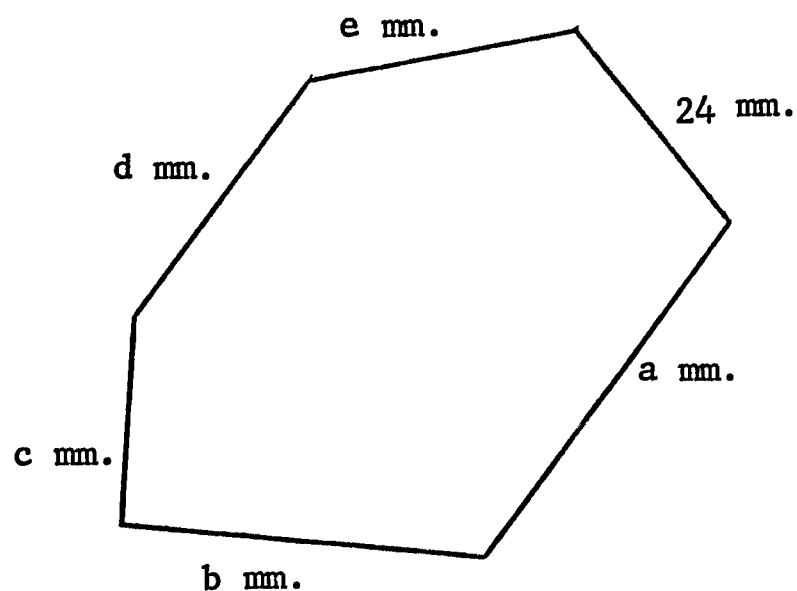
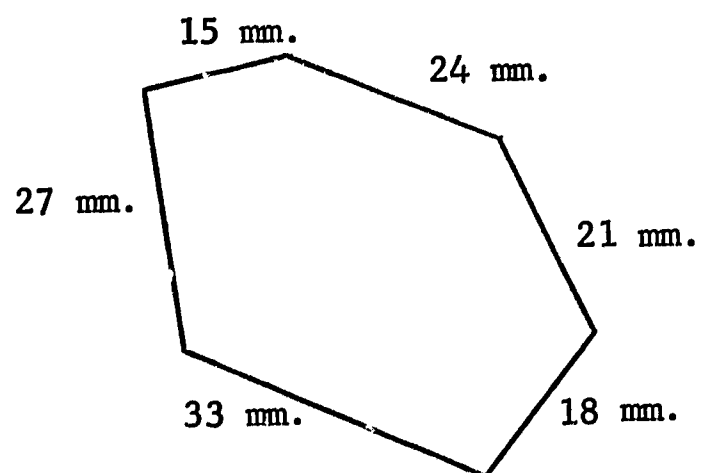
b = _____

c = _____

d = _____

e = _____

5.



a = _____

b = _____

c = _____

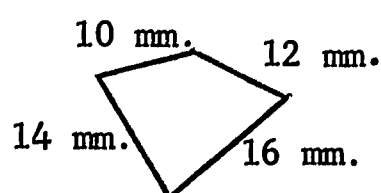
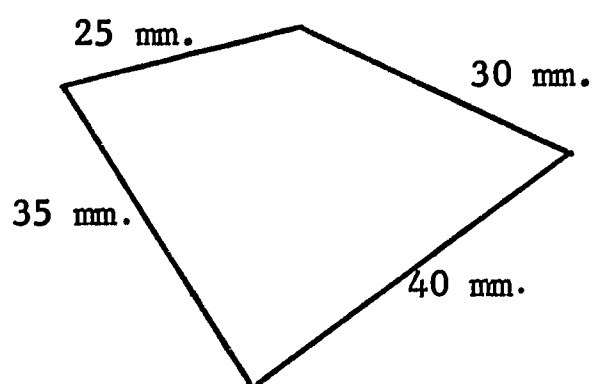
d = _____

e = _____

- 6) 5, 7, 12, and 8 in.
 7) 12, 19, 23, 17, and 15 in.
 8) 3, 9, 8, 4, and 11 ft.
 9) 20, 25, 35, and 40 in.
 10) 12, 36, 54, 72, and 60 in.

- 20, _____, _____, and _____ in.
 36, _____, _____, _____, and _____ in.
 15, _____, _____, _____, and _____ ft.
 16, _____, _____, and _____ ft.
 10, _____, _____, _____, and _____ in.

Do you recall that the perimeter means the distance around? A triangle with sides of lengths 3, 4 and 5 centimeters has a perimeter of 12 centimeters. There is a very interesting relationship between the perimeters of similar polygons. Study the pairs of similar polygons represented here and see if you can determine the relationship.

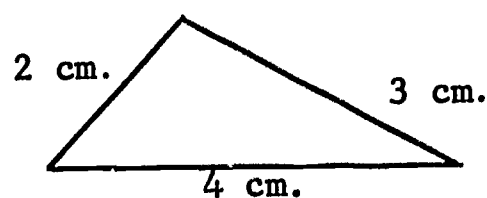
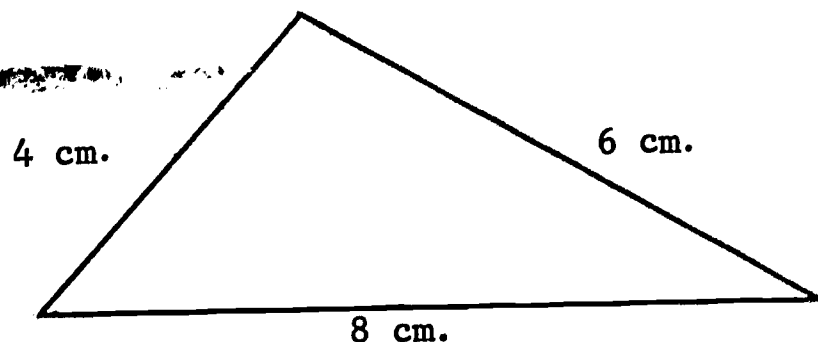


Correspondences	Fractions
$25 \longleftrightarrow 10$	$\frac{10}{25} = ?$
$30 \longleftrightarrow 12$	$\frac{12}{30} = ?$
$40 \longleftrightarrow 16$	$\frac{16}{40} = ?$
$35 \longleftrightarrow 14$	$\frac{14}{35} = ?$

The perimeters of the two quadrilaterals are 130 mm ($25 + 30 + 40 + 35$) and 52 mm ($10 + 12 + 16 + 14$). Compare the two perimeters in the form of a fraction, $\frac{52}{130}$. What fraction with a denominator of 5 is this equivalent to? _____ (Hint: $\frac{52}{130} = \frac{?}{5}$)

In each of the problems 1 - 5 set up the correspondences among the lengths of the sides of the two polygons, form the fractions, and then compare these fractions with the fraction formed by the perimeters of the two polygons.

1.

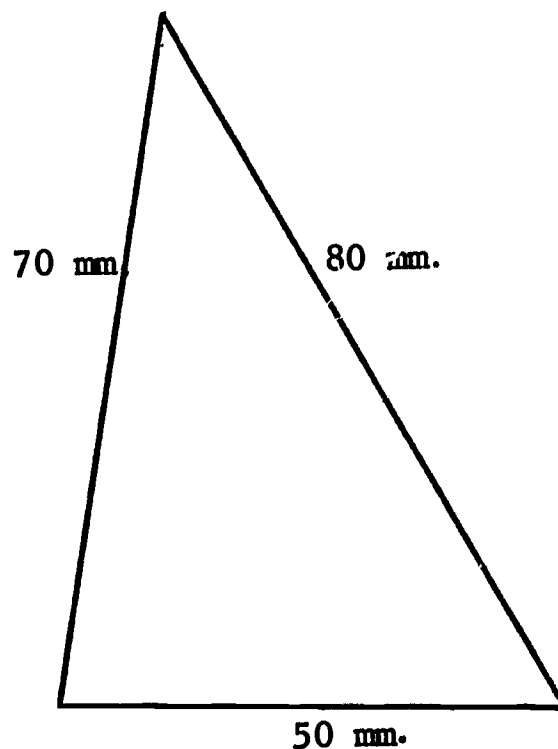
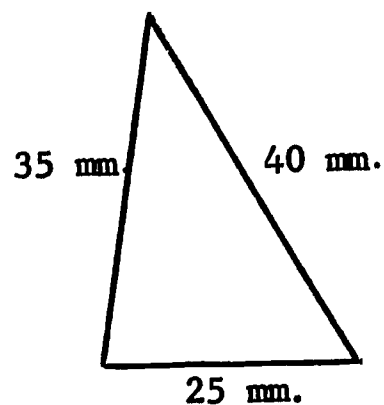


Correspondences	Fractions
$4 \longleftrightarrow 2$	$\frac{2}{4}$
$8 \longleftrightarrow 4$	$\frac{4}{8}$
$6 \longleftrightarrow 3$	$\frac{3}{6}$
$18 \longleftrightarrow 9$	$\frac{9}{18}$

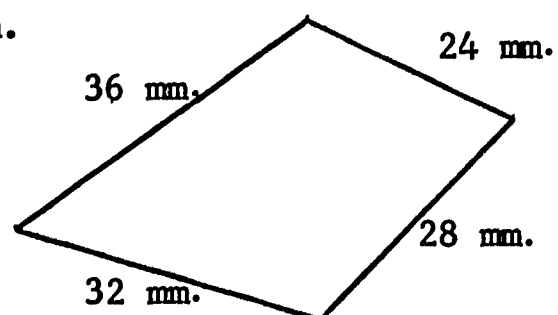
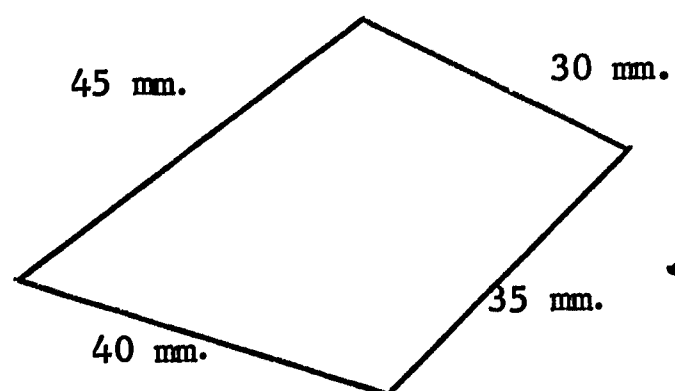
Perimeter of first triangle: $(8 + 6 + 4)\text{cm} = 18\text{ cm}$

Perimeter of second triangle: $(4 + 3 + 2)\text{cm} = 9\text{ cm}$

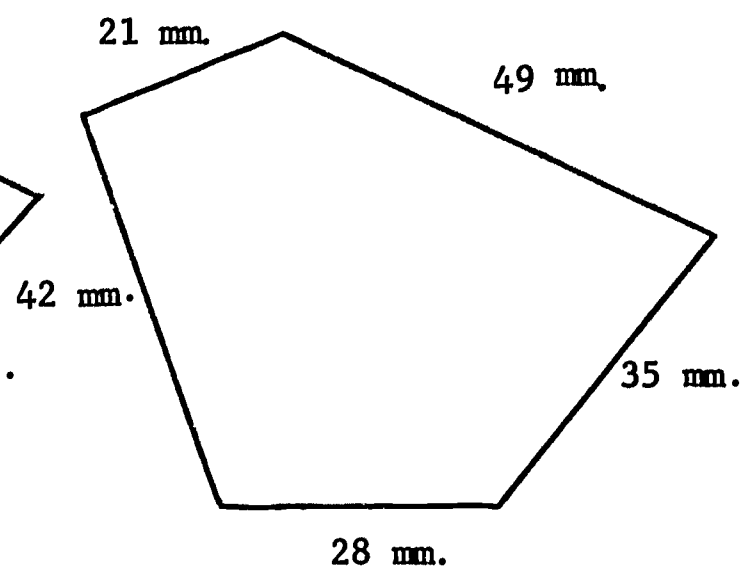
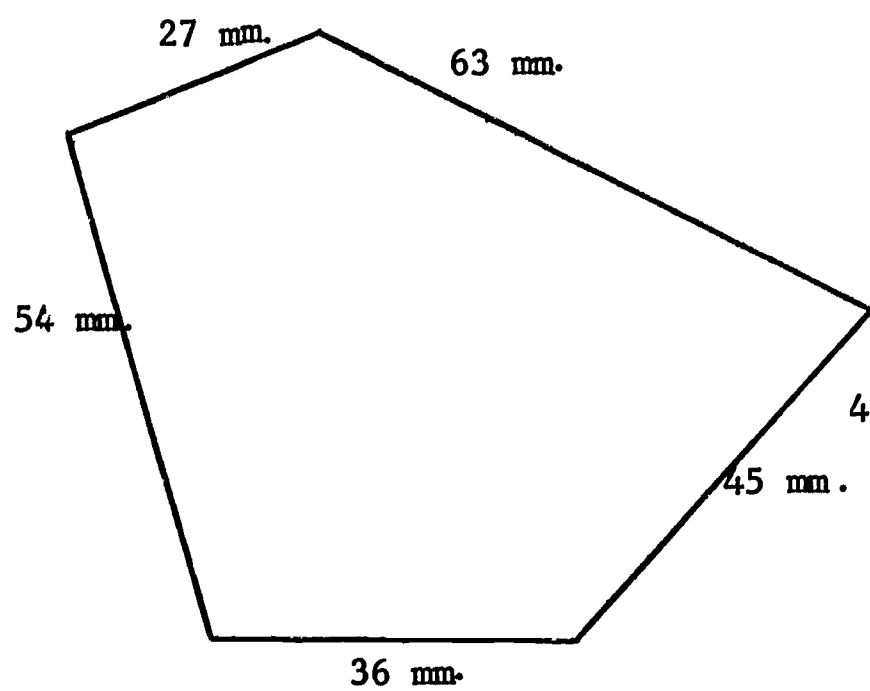
2.



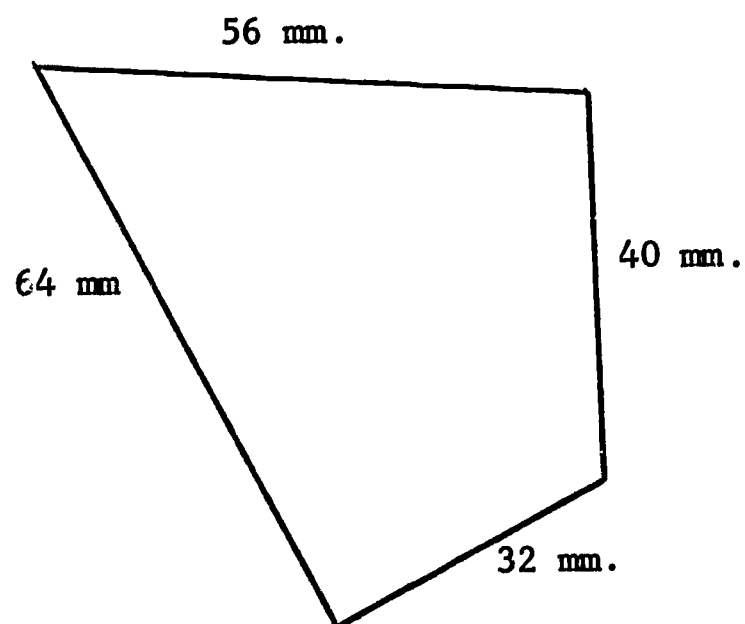
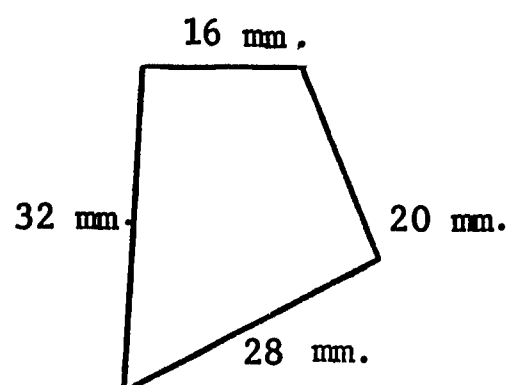
3.



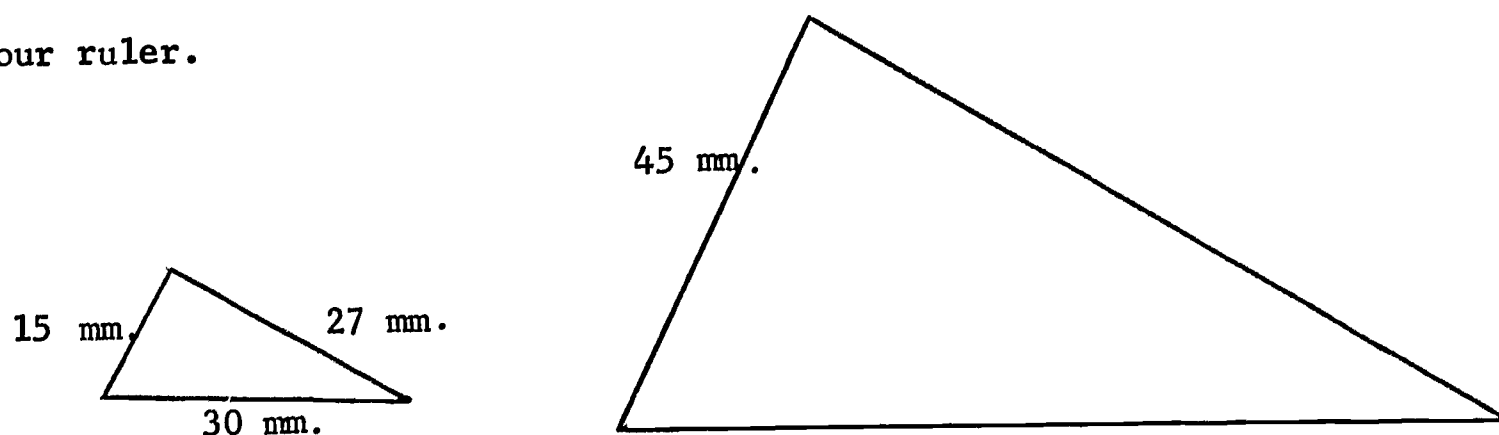
4.



5.



Example: These two triangles are similar. The side of length 15 mm in the first triangle corresponds to the side of length 45 mm in the second triangle. Find the perimeter of the second triangle without using your ruler.



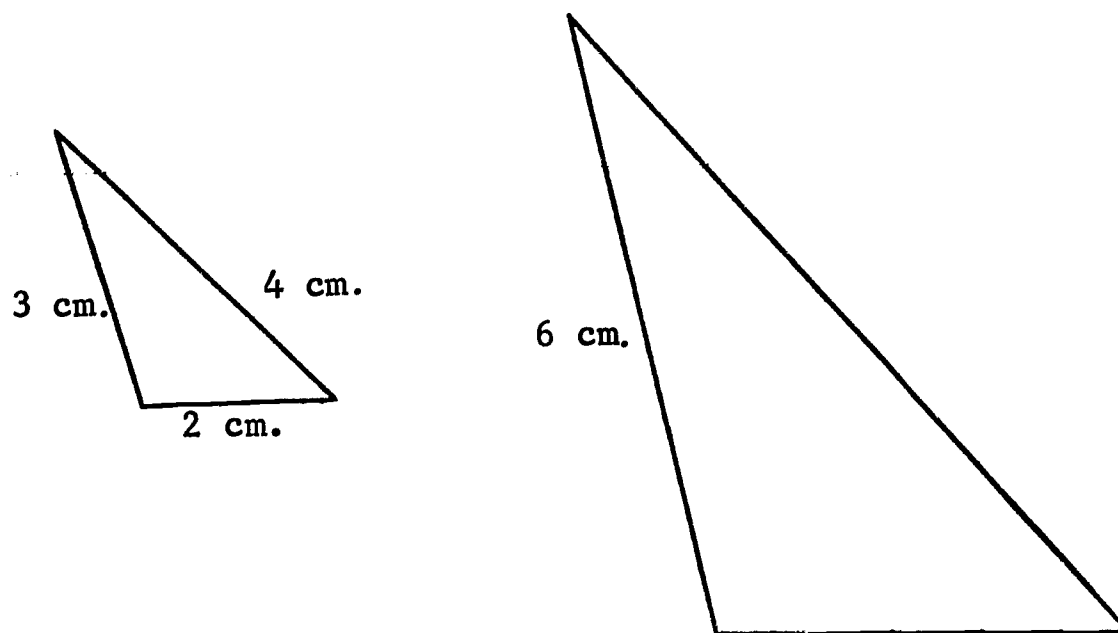
The perimeter of first triangle is $(15 + 30 + 27)$ mm, or 72 mm. Therefore:

$$\frac{15}{45} = \frac{72}{?}$$

Activities

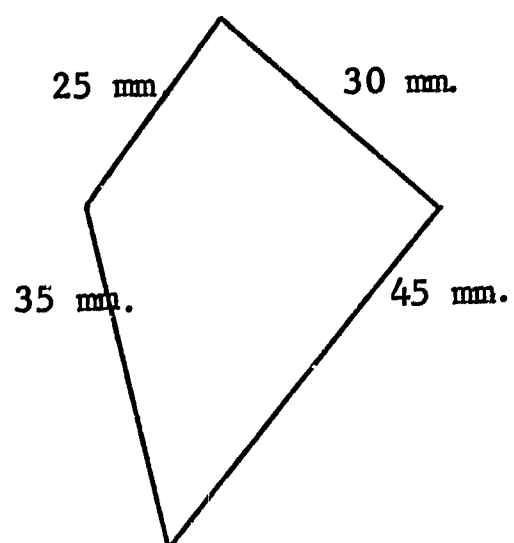
Find, without using your ruler, the perimeter of the second polygon in each of these problems. The two polygons in each problem are similar.

1.

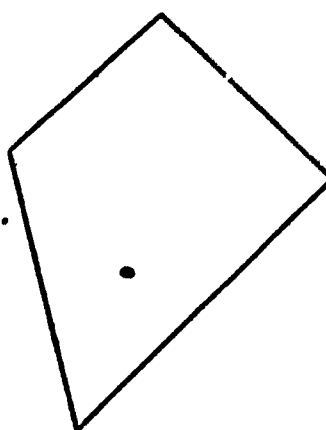


(Hint: $\frac{3}{6} = \frac{9}{?}$)

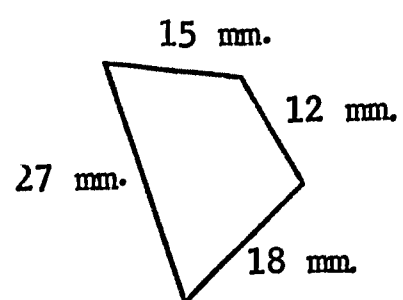
2.



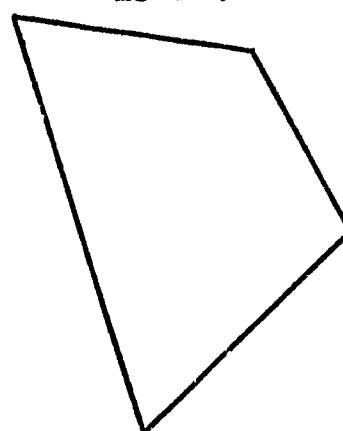
28 mm.



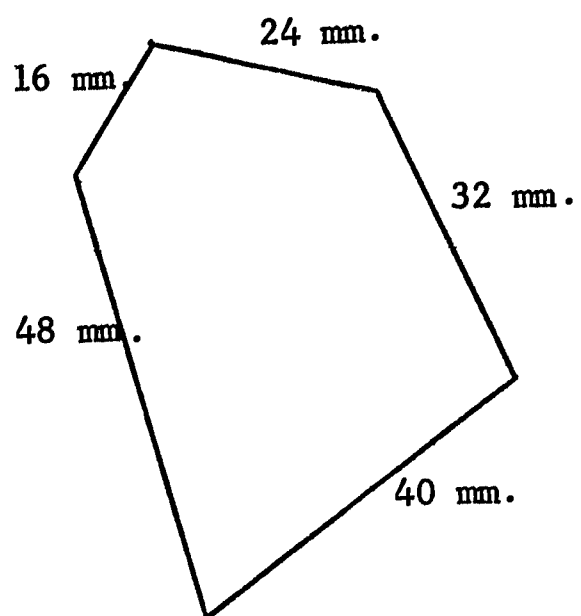
3.



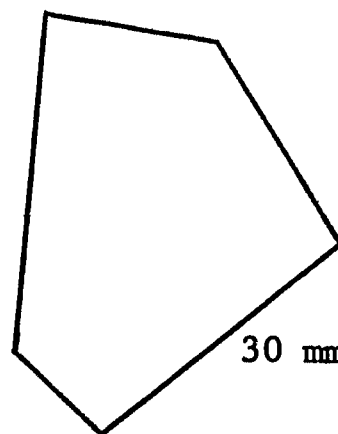
25 mm.



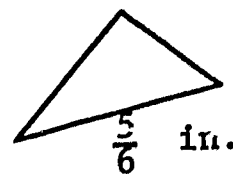
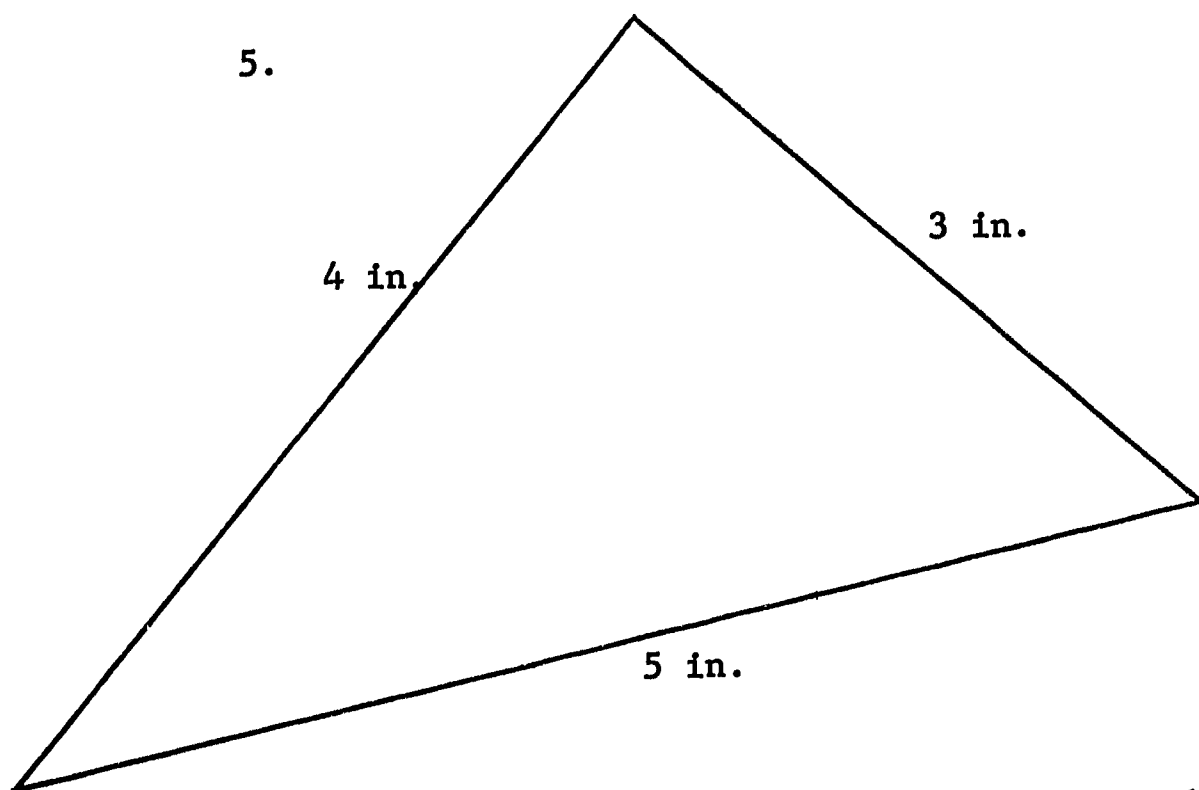
4.



30 mm.



5.



In each of the problems 6 - 10 the lengths of the sides of a polygon are given, in inches, along with the length of the side of a second similar polygon. Find the perimeter of the second polygon.

6. 4
7
12 \longleftrightarrow 48
15

7. 5
10
30
25 \longleftrightarrow 5

8. 12
48
36
60 \longleftrightarrow 40
72

9. 3
4 \longleftrightarrow 16
5

10. 10
20 \longleftrightarrow 4
30
60
70
20